AKADEMIA EDUKACYJNO - INŻYNIERSKA

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MATURA STUDIA PRAKTYKA STAŻ PRACA BIZNES PRZEMYSŁ STEAM

KOMPLEKSOWE WSPARCIE EDUKACYJNE **ON – LINE** NA KAŻDYM ETAPIE KSZTAŁCENIA INŻYNIERSKIEGO

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ANALIZA MATEMATYCZNA I

"CAŁKA I - POLE OBSZARU OGRANICZONEGO KRZYWYMI"

PROJEKT

Oblicz:

– całkę pojedynczą

interpretowaną jako pole obszaru ograniczonego krzywymi.

DANE:

$\int = OK$

SZUKAM:

$\int = ?$

1) $$\int \frac{1}{x^4}\,dx = \int x^{-4}\,dx = -\frac{1}{3x^3} + C$$

2) $$\int \frac{\sqrt{x}}{x^5}\,dx = \int \left(x^{\frac{1}{2}} \cdot x^{-5}\right) dx = \int x^{\left(\frac{1}{2} - \frac{10}{2}\right)} dx = \int x^{-\frac{9}{2}}\,dx = \frac{x^{-\frac{7}{2}}}{-\frac{7}{2}} = -\frac{2}{7x^{\frac{7}{2}}} = -\frac{2}{7x^3\sqrt{x}} + C$$

3) $$\int \sqrt[3]{x}\,dx = \int x^{\frac{1}{3}}\,dx = \frac{x^{\frac{4}{3}}}{\frac{4}{3}} = \frac{3}{4}\sqrt[3]{x^4} = \frac{3}{4}x\sqrt[3]{x} + C$$

4) $$\int \frac{x}{\sqrt[3]{x}}\,dx = \int x : x^{\frac{1}{3}}\,dx = \int x^{\frac{2}{3}}\,dx = \frac{3}{5}x^{\frac{5}{3}} = \frac{3}{5}x\sqrt[3]{x^2} + C$$

5) $$\int \frac{1}{\sqrt[4]{x^3}}\,dx = \int x^{-\frac{3}{4}}\,dx = 4x^{\frac{1}{4}} = 4\sqrt[4]{x} + C$$

6) $$\int \frac{\sqrt[4]{x^3}}{x^4}\,dx = \int \left(x^{\frac{3}{4}} : x^4\right) dx = \int x^{\frac{3-16}{4}}\,dx = \int x^{-\frac{13}{4}}\,dx = -\frac{4}{9}x^{-\frac{9}{4}} = -\frac{4}{9\sqrt[4]{x^9}} = -\frac{4}{9x^2\sqrt[4]{x}} + C$$

7) $\displaystyle\int \frac{x\sqrt{x}}{\sqrt[5]{x^4}}\,dx = \int \left(x^{\frac{3}{2}} : x^{\frac{4}{5}}\right) dx = \int x^{\frac{15-8}{10}}\,dx = \int x^{\frac{7}{10}}\,dx = \frac{10}{17}\,x^{\frac{17}{10}} = \frac{10}{17}\,x\sqrt{x^7} + C$

8) $\displaystyle\int \frac{\sqrt{x}\cdot\sqrt[3]{x}}{x\cdot\sqrt[4]{x}}\,dx = \int \frac{x^{\frac{1}{2}}\cdot x^{\frac{1}{3}}}{x\cdot x^{\frac{1}{4}}}\,dx = \int \frac{x^{\frac{5}{6}}}{x^{\frac{5}{4}}}\,dx = \int x^{\left(\frac{5}{6}-\frac{5}{4}\right)}\,dx = \int x^{-\frac{5}{12}}\,dx = \frac{12}{7}\,x^{\frac{7}{12}} = \frac{12}{7}\sqrt[12]{x^7} + C$

9) $\displaystyle\int (5x^4 + 9x^3 - 7x^2 + 3x - 4)\,dx = 5\int x^4\,dx + 9\int x^3\,dx - 7\int x^2\,dx + 3\int x\,dx - 4\int dx =$

$\displaystyle = \cancel{5}\cdot\frac{x^5}{\cancel{5}} + 9\cdot\frac{x^4}{4} - 7\,\frac{x^3}{3} + 3\,\frac{x^2}{2} - 4x = x^5 + \frac{9}{4}x^4 - \frac{7}{3}x^3 + \frac{3}{2}x^2 - 4x + C$

10) $\displaystyle\int e^{-2x}\,dx =$

$-2x = t \quad /\,()'$

$-2dx = dt$

$dx = -\frac{1}{2}\,dt$

$\displaystyle = -\frac{1}{2}\int e^t\,dt = -\frac{1}{2}e^t = -\frac{1}{2}e^{-2x} + C$

11) $\displaystyle\int e^{4x-5}\,dx =$

$4x - 5 = t \quad /\,()'$

$4\,dx = dt$

$dx = \frac{1}{4}\,dt$

$\displaystyle = \frac{1}{4}\int e^t\,dt = \frac{1}{4}e^t = \frac{1}{4}e^{4x-5} + C$

12) $\int (2x+3)^9\,dx = $ $\quad 2x+3 = t \;/\;()' \quad 2\,dx = dt \quad dx = \frac{1}{2}\,dt$ $\quad = \frac{1}{2}\int t^9\,dt = \frac{1}{2}\cdot\frac{t^{10}}{10} = \frac{1}{20}(2x+3)^{10} + C$

13) $\int \frac{1}{(2x-3)^4}\,dx = $ $\quad 2x-3 = t \;/\;()' \quad 2\,dx = dt \quad dx = \frac{1}{2}\,dt$ $\quad = \frac{1}{2}\int \frac{dt}{t^4} = \frac{1}{2}\int t^{-4}\,dt = \frac{1}{2}\cdot\frac{t^{-3}}{-3} = -\frac{1}{6t^3} = -\frac{1}{6(2x-3)^3} + C$

14) $\int \frac{1}{5x+4}\,dx = $ $\quad 5x+4 = t \;/\;()' \quad 5\,dx = dt \quad dx = \frac{1}{5}\,dt$ $\quad = \frac{1}{5}\int \frac{dt}{t} = \frac{1}{5}\ln|t| = \frac{1}{5}\ln|5x+4| + C$

15) $\int \frac{1}{2-x}\,dx = $ $\quad 2-x = t \;/\;()' \quad -dx = dt \quad dx = -dt$ $\quad = -\int \frac{dt}{t} = -\ln|t| = -\ln|2-x| = -\ln|-x+2| + C$

16) $\int \cos 5x\,dx = $ $\quad 5x = t \;/\;()' \quad 5\,dx = dt \quad dx = \frac{1}{5}\,dt$ $\quad = \frac{1}{5}\int \cos t\,dt = \frac{1}{5}\sin t = \frac{1}{5}\sin 5x + C$

17)

$$\int \sin(2x+1)\,dx = \quad \begin{aligned} 2x+1 &= t \quad /\,(\,)' \\ 2\,dx &= dt \\ dx &= \tfrac{1}{2}\,dt \end{aligned} \quad = \frac{1}{2}\int \sin t\,dt = -\frac{1}{2}\cos t = -\frac{1}{2}\cos(2x+1) + C$$

18)

$$\int \frac{e^x}{e^x+4}\,dx = \quad \begin{aligned} e^x+4 &= t \quad /\,(\,)' \\ e^x\,dx &= dt \end{aligned} \quad = \int \frac{dt}{t} = \ln|t| = \ln|e^x+4| + C$$

19)

$$\int x e^{-x^2}\,dx = \quad \begin{aligned} -x^2 &= t \quad /\,(\,)' \\ -2x\,dx &= dt \\ x\,dx &= -\tfrac{1}{2}\,dt \end{aligned} \quad = -\frac{1}{2}\int e^t\,dt = -\frac{1}{2}e^t = -\frac{1}{2}e^{-x^2} + C$$

20)

$$\int \frac{3x^2}{x^3+2}\,dx = \quad 3x^2 \quad \begin{aligned} x^3+2 &= t \quad /\,(\,)' \\ 3x^2\,dx &= dt \end{aligned} \quad = \int \frac{dt}{t} = \ln|t| = \ln|x^3+2| + C$$

21)

$$\int \frac{1}{\sqrt[3]{4x+3}}\,dx = \quad \begin{aligned} 4x+3 &= t \quad /\,(\,)' \\ 4\,dx &= dt \\ dx &= \tfrac{1}{4}\,dt \end{aligned} \quad = \frac{1}{4}\int \frac{dt}{\sqrt[3]{t}} = \frac{1}{4}\int t^{-\frac{1}{3}}\,dt = \frac{1}{4}\cdot\frac{3}{2}\,t^{\frac{2}{3}} = \frac{3}{8}\sqrt[3]{(4x+3)^2} + C$$

22)

$$\int \frac{\sin x}{4-\cos x}\,dx = \quad \begin{aligned} 4-\cos x &= t \quad /\,(\,)' \\ \sin x\,dx &= dt \end{aligned} \quad = \int \frac{dt}{t} = \ln|t| = \ln|-\cos x + 4| + C$$

23) $\int \frac{\sin 2x}{\sin^2 x + 3}\,dx =$ $\quad \sin^2 x + 3 = t$, $\; 2\sin x \cos x\,dx = dt$ $\quad \sin^2 x + \cos^2 x = 1$, $\; \sin 2x = 2\sin x\cos x$, $\; \cos 2x = \cos^2 x - \sin^2 x$ $\quad = \int \frac{2\sin x\cos x}{\sin^2 x + 3}\,dx =$

$$= \int \frac{dt}{t} = \ln|t| = \ln|\sin^2 x + 3| + C$$

24) $\int \frac{3x^2 + 6x - 4}{x}\,dx = 3\int x\,dx + 6\int dx - 4\int \frac{dx}{x} = 3\frac{x^2}{2} + 6x - 4\ln|x| = \frac{3}{2}x^2 + 6x - 4\ln|x| + C$

25) $\int \mathrm{ctg}^2 x\,dx =$ $\int \frac{\cos^2 x}{\sin^2 x}\,dx =$ $\quad \sin^2 x + \cos^2 x = 1$, $\; \cos^2 x = 1 - \sin^2 x$ $\quad = \int \frac{1 - \sin^2 x}{\sin^2 x}\,dx = \int \frac{1}{\sin^2 x}\,dx - \int 1\,dx =$

$$= -\mathrm{ctg}\,x - x + C$$

26) $\int e^x \cos x\,dx =$ $\quad f = e^x$, $f' = e^x$ $\quad g' = \cos x$, $g = \sin x$ $\quad = e^x \sin x - \int e^x \sin x\,dx$ 1)

1) $\int e^x \sin x\,dx =$ $\quad f = e^x$, $f' = e^x$ $\quad g' = \sin x$, $g = -\cos x$ $\quad = -e^x\cos x + \int e^x \cos x\,dx$ 4)

11) $\int e^x \cos x\,dx = \quad e^x \sin x - \int e^x \sin x\,dx = e^x \sin x - \left(-e^x \cos x + \int e^x \cos x\,dx\right) = e^x(\sin x + \cos x) - \int e^x \cos x\,dx$

$\Rightarrow \quad 2\int e^x \cos x\,dx = e^x(\sin x + \cos x) \quad /:2$

$\Rightarrow \quad \int e^x \cos x\,dx = \frac{1}{2} e^x(\sin x + \cos x) + C$

24) $\int \frac{x}{\cos^2 x}\,dx =$

$f = x \qquad g' = \frac{1}{\cos^2 x} \qquad = x\,tg\,x - \int tg\,x\,dx$ 1)

$f' = 1 \qquad g = tg\,x$

1) $\int tg\,x\,dx = \int \frac{\sin x}{\cos x}\,dx =$

$\cos x = t$

$-\sin x\,dx = dt$

$\sin x\,dx = -dt$

$= -\int \frac{dt}{t} = -\ln|t| = -\ln|\cos x| + C$

$\Rightarrow \quad \int \frac{x}{\cos^2 x}\,dx = x\,tg\,x + \ln|\cos x| + C$

28) $\int x^5 \ln x\,dx =$

$f = \ln x \qquad g' = x^5$

$f' = \frac{1}{x} \qquad g = \frac{1}{6}x^6$

$= \frac{1}{6}x^6 \ln|x| - \frac{1}{6}\int x^5\,dx = \frac{1}{6}x^6 \ln|x| - \frac{1}{36}x^6 + C$

28) $\int \frac{\ln x}{x^4}\,dx =$

$f = \ln x \qquad g' = x^{-4}$

$f' = \frac{1}{x} \qquad g = -\frac{1}{3x^3}$

$= -\frac{\ln x}{3x^3} + \frac{1}{3}\int x^{-4}\,dx = -\frac{\ln x}{3x^3} - \frac{1}{9x^3} = -\frac{1}{3x^3}\left(\ln x + \frac{1}{3}\right) + C$

30) $\int x \ln^2 x \, dx =$ $\quad f = \ln^2 x \quad g' = x$, $\quad f' = 2\frac{\ln x}{x} \quad g = \frac{1}{2}x^2$ $\quad = \frac{1}{2}x^2\ln^2 x - \int x \ln x \, dx$ 1)

1) $\int x \ln x \, dx =$ $\quad f = \ln x \quad g' = x$, $\quad f' = \frac{1}{x} \quad g = \frac{1}{2}x^2$ $\quad = \frac{1}{2}x^2 \ln x - \frac{1}{2}\int x \, dx = \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$

$$\Rightarrow \int x \ln^2 x \, dx = \frac{1}{2}x^2\ln^2 x - \frac{1}{2}x^2 \ln x + \frac{1}{4}x^2 = \frac{1}{2}x^2\left(\ln^2 x - \ln x + \frac{1}{2}\right) + C$$

31) $\int \frac{1}{(x+1)^2} dx =$ $\quad x + 1 = t$, $\quad dx = dt$ $\quad = \int \frac{dt}{t^2} = \int t^{-2} dt = -\frac{1}{t} = -\frac{1}{x+1} + C$

32) $\int \frac{1}{(x-5)^3} dx =$ $\quad x - 5 = t$, $\quad dx = dt$ $\quad = \int \frac{dt}{t^3} = \int t^{-3} dt = -\frac{1}{2t^2} = -\frac{1}{2(x-5)^2} + C$

33) $\int \sqrt{x}\, e^{\sqrt{x}} dx =$ $\quad \sqrt{x} = t$, $\quad x = t^2$, $\quad dx = 2t\,dt$ $\quad = 2\int t^2 e^t dt$ 1)

1) $\int t^2 e^t dt =$ $\quad f = t^2 \quad g' = e^t$, $\quad f' = 2t \quad g = e^t$ $\quad = t^2 e^t - 2\int t e^t dt$ 11)

11) $\int t e^t dt =$ $\quad f = t \quad g' = e^t$, $\quad f' = 1 \quad g = e^t$ $\quad = t e^t - \int e^t dt = t e^t - e^t + C$

$$= \int \sqrt{x}\, e^{\sqrt{x}}\, dx = 2(t^2 e^t - 2(te^t - e^t)) = 2t^2e^t - 4te^t + 4e^t = 2e^t(t^2 - 2t + 2) = 2e^{\sqrt{x}}(x - 2\sqrt{x} + 2) + C$$

34)
$$\int \frac{\ln^3 x}{x}\, dx = \int x^{-1} \ln^3 x\, dx = \quad \begin{matrix} f = \ln^3 x & g' = \frac{1}{x} \\ f' = 3\ln^2 x \cdot \frac{1}{x} & g = \ln x \end{matrix} \quad = \ln^4 x - 3\int \frac{\ln^3 x}{x}\, dx + C$$

*
$$\Rightarrow 4\int \frac{\ln^3 x}{x}\, dx = \ln^4 x + C \quad /:4$$

$$\Rightarrow \int \frac{\ln^3 x}{x}\, dx = \frac{1}{4}\ln^4 x + C$$

35)
$$\int \frac{\sqrt{\ln x}}{x}\, dx = \int x^{-1}(\ln x)^{\frac{1}{2}}\, dx = \quad \begin{matrix} f = (\ln x)^{\frac{1}{2}} & g' = \frac{1}{x} \\ f' = \frac{1}{2}(\ln x)^{-\frac{1}{2}} \cdot \frac{1}{x} & g = \ln x \end{matrix} \quad = (\ln x)^{\frac{1}{2}} \cdot \ln x - \frac{1}{2}\int \frac{\sqrt{\ln x}}{x}\, dx$$

*
$$\Rightarrow \int \frac{\sqrt{\ln x}}{x}\, dx = (\ln x)^{\frac{3}{2}} - \frac{1}{2}\int \frac{\sqrt{\ln x}}{x}\, dx$$

$$\Rightarrow \frac{3}{2}\int \frac{\sqrt{\ln x}}{x}\, dx = (\ln x)^{\frac{3}{2}} \quad /\cdot \frac{2}{3} \qquad \Rightarrow \int \frac{\sqrt{\ln x}}{x}\, dx = \frac{2}{3}(\ln x)^{\frac{3}{2}} + C$$

36)
$$\int \frac{1}{x(\ln x - 4)}\, dx = \quad \begin{matrix} \ln x - 4 = t \\ \frac{dx}{x} = dt \end{matrix} \quad = \int \frac{dt}{t} = \ln|t| = \ln|\ln x - 4| + C$$

37) $\int \frac{x}{\sqrt{x^2+4}}\,dx =$ $\begin{aligned}\sqrt{x^2+4} &= t\\ x^2+4 &= t^2\\ 2x\,dx &= 2t\,dt\\ x\,dx &= t\,dt\end{aligned}$ $= \int \frac{\cancel{t}\,dt}{\cancel{t}} = \int dt = t = \sqrt{x^2+4} + C$

38) $\int e^{\cos x} \sin x\,dx =$ $\begin{aligned}\cos x &= t\\ -\sin x\,dx &= dt\\ \sin x\,dx &= -dt\end{aligned}$ $= -\int e^t\,dt = -e^t = -e^{\cos x} + C$

39) $\int \frac{\ln x}{\sqrt{x}}\,dx = \int x^{-\frac{1}{2}} \ln x\,dx =$ $\begin{aligned}f &= \ln x \quad & g' &= x^{-\frac{1}{2}}\\ f' &= \frac{1}{x} \quad & g &= 2x^{\frac{1}{2}}\end{aligned}$ $= 2\sqrt{x}\ln x - 2\int x^{-\frac{1}{2}}dx = 2\sqrt{x}(\ln x - 2) + C$

40) $\int x\sqrt{x}\,\ln x\,dx = \int x^{\frac{3}{2}} \ln x\,dx =$ $\begin{aligned}f &= \ln x \quad & g' &= x^{\frac{3}{2}}\\ f' &= \frac{1}{x} \quad & g &= \frac{2}{5}x^{\frac{5}{2}}\end{aligned}$ $= \frac{2}{5}x^{\frac{5}{2}}\ln x - \frac{2}{5}\int x^{\frac{3}{2}}dx = \frac{2}{5}x^{\frac{5}{2}}\ln x - \frac{4}{25}x^{\frac{5}{2}} =$

$= \frac{2}{5}\sqrt{x^5}\left(\ln x - \frac{2}{5}\right) + C$

41) $\int (e^{x^2} + 5e^x)\,dx = \int e^x(e^x+5)\,dx =$ $\begin{aligned}e^x + 5 &= t\\ e^x\,dx &= dt\end{aligned}$ $= \int t\,dt = \frac{1}{2}t^2 = \frac{1}{2}(e^x+5)^2 + C$

42) $\int x(3x^2-2)^{19}dx =$ $\quad 3x^2-2=t$, $6x\,dx = dt$, $x\,dx = \frac{1}{6}dt$ $\quad = \frac{1}{6}\int t^{19}dt = \frac{1}{6}\cdot\frac{t^{20}}{20} = \frac{1}{120}(3x^2-2)^{20}+C$

43) $\int \frac{1}{(x+3)\ln|x+3|}dx =$ $\quad \ln|x+3| = t$, $\frac{1}{x+3}dx = dt$ $\quad = \int\frac{dt}{t} = \ln|t| = \ln|\ln|x+3||+C$

44) $\int \frac{2x-3}{x^2-3x+4}dx =$ $\quad x^2-3x+4=t$, $(2x-3)dx = dt$ $\quad = \int\frac{dt}{t} = \ln|t| = \ln|x^2-3x+4|+C$

45) $\int \frac{1}{(x-3)(x+5)}dx =$

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$$\frac{1}{(x-3)(x+5)} = \frac{A}{(x-3)} + \frac{B}{(x+5)} \quad /\cdot(x-3)(x+5)$$

$$1 = A(x+5) + B(x-3)$$
$$1 = Ax + 5A + Bx - 3B$$
$$1 = (A+B)x^1 + (5A-3B)x^0$$
$$0x^1 + 1x^0 = (A+B)x^1 + (5A-3B)x^0$$

$$\begin{cases} \text{I)}\ A+B=0 \quad /\cdot 3 \\ \text{II)}\ 5A-3B=1 \end{cases}$$

$$\begin{cases} \text{I)}\ 3A+3B=0 \\ \text{II)}\ 5A-3B=1 \end{cases} \quad \oplus$$

$$3A + \cancel{3B} + 5A - \cancel{3B} = 1$$
$$8A = 1$$
$$A = \frac{1}{8}$$

I) $A + B = 0$

$$B = -A = -\frac{1}{8}$$

$$= \int\frac{1}{(x-3)(x+5)}dx = \frac{1}{8}\int\frac{dx}{(x-3)} + $$
$$+ -\frac{1}{8}\int\frac{dx}{(x+5)} = \frac{1}{8}\ln\left|\frac{x-3}{x+5}\right| + C$$

46) $\int \frac{x+2}{x^2-5x+6} dx =$ **

* $x^2 - 5x + 6 = 0$

$\Delta = B^2 - 4AC$

$\Delta = 25 - 24$

$\Delta = 1$

$\sqrt{\Delta} = 1$

$$\begin{cases} x_1 = \frac{-B - \sqrt{\Delta}}{2A} = \frac{5 - \sqrt{1}}{2} = \frac{4}{2} = 2 \\ x_2 = \frac{-B + \sqrt{\Delta}}{2A} = \frac{5 + \sqrt{1}}{2} = \frac{6}{2} = 3 \end{cases}$$

$\Rightarrow x^2 - 5x + 6 = (x-2)(x-3)$

**

$$\frac{x+2}{(x-2)(x-3)} = \frac{A}{(x-2)} + \frac{B}{(x-3)} \quad / \cdot (x-2)(x-3)$$

$x + 2 = A(x-3) + B(x-2)$

$x + 2 = Ax - 3A + Bx - 2B$

$x + 2 = (A+B)x^1 + (-3A - 2B)x^0$

$1x^1 + 2x^0 = (A+B)x^1 + (-3A-2B)x^0$ $\oplus$

$$\begin{cases} ^{I)}\ A + B = 1 \quad / \cdot 2 \\ ^{II)}\ -3A - 2B = 2 \end{cases}$$ $\oplus$

$$\begin{cases} ^{I)}\ 2A + 2B = 2 \\ ^{II)}\ -3A - 2B = 2 \end{cases}$$ $\oplus$

$2A + \cancel{2B} - 3A - \cancel{2B} = 2 + 2$

$-A = 4 \quad / \cdot (-1)$

$A = -4$

$^{I)}\ A + B = 1$

$B = 1 - A$

$B = 1 - (-4)$

$B = 5$

$$\Rightarrow \begin{cases} A = -4 \\ B = 5 \end{cases}$$

$$\Rightarrow \int \frac{x+2}{x^2-5x+6} dx = -4\int \frac{dx}{(x-2)} + 5\int \frac{dx}{(x-3)} = 5 \ln|x-3| - 4 \ln|x-2| + C$$

44) $\int \frac{x^2}{x^2-x-2}\,dx =$ *

*

$$\frac{1}{(x^2):(x^2-x-2)}$$
$$\frac{-x^2+x+2}{= x+2}$$

$$\Rightarrow \int \frac{x^2}{x^2-x-2}\,dx = \int 1\,dx + \int \frac{x+2}{x^2-x-2}\,dx \quad 1)$$

1) $\int \frac{x+2}{x^2-x-2}\,dx =$ **

*

$$x^2 - x - 2 = 0$$

$$\Delta = B^2 - 4AC$$
$$\Delta = 1 + 8$$
$$\Delta = 9$$
$$\sqrt{\Delta} = 3$$

$$\begin{cases} x_1 = \frac{-B-\sqrt{\Delta}}{2A} = \frac{1-3}{2} = \frac{-2}{2} = -1 \\ x_2 = \frac{-B+\sqrt{\Delta}}{2A} = \frac{1+3}{2} = \frac{4}{2} = 2 \end{cases}$$

$$\Rightarrow x^2 - x - 2 = (x+1)(x-2)$$

**

$$\frac{x+2}{x^2-x-2} = \frac{x+2}{(x+1)(x-2)} = \frac{A}{(x+1)} + \frac{B}{(x-2)} \quad /\cdot (x+1)(x-2)$$

$$x + 2 = A(x-2) + B(x+1)$$
$$x + 2 = Ax - 2A + Bx + B$$
$$x + 2 = (A+B)x - 2A + B$$
$$1x^1 + 2x^0 = (A+B)x^1 + (-2A+B)x^0$$

$$\begin{cases} 1)\ A + B = 1 \quad /\cdot 2 \\ 2)\ -2A + B = 2 \end{cases}$$

$$\begin{cases} 2A + 2B = 2 \\ -2A + B = 2 \end{cases} \oplus$$

$$\cancel{2A} + 2B - \cancel{2A} + B = 2 + 2$$
$$3B = 4 \quad /:3$$
$$B = \frac{4}{3}$$

1) $A + B = 1$

$$A = 1 - B$$
$$A = 1 - \frac{4}{3} = -\frac{1}{3}$$

$$\Rightarrow \begin{cases} A = -\frac{1}{3} \\ B = \frac{4}{3} \end{cases}$$

$$\Rightarrow \int \frac{x^2}{x^2-x-2}\,dx = \int 1\,dx - \frac{1}{3}\int \frac{dx}{(x+1)} + \frac{4}{3}\int \frac{dx}{(x-2)} =$$

$$= x + \frac{4}{3}\ln|x-2| - \frac{1}{3}\ln|x+1| + C$$

48) $\int \frac{-4x+12}{x^2(x+6)}\,dx =$ *

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$$\frac{-4x+12}{x^2(x+6)} = \frac{Ax+B}{x^2} + \frac{C}{(x+6)} \quad / \cdot x^2(x+6)$$

$$-4x+12 = (Ax+B)(x+6) + Cx^2$$

$$-4x+12 = Ax^2 + 6Ax + Bx + 6B + Cx^2$$

$$-4x+12 = (A+C)x^2 + (6A+B)x + 6B$$

$$0x^2 - 4x^1 + 12x^0 = (A+C)x^2 + (6A+B)x^1 + 6Bx^0$$

$$\begin{cases} \text{I)}\ A + C = 0 \\ \text{II)}\ 6A + B = -4 \\ \text{III)}\ 6B = 12 \end{cases}$$

III)

$$6B = 12$$

$$B = 2$$

II)

$$6A + B = -4$$

$$6A + 2 = -4$$

$$6A = -6 \quad /:6$$

$$A = -1$$

I)

$$A + C = 0$$

$$C = -A$$

$$C = -(-1) = 1$$

$$\Rightarrow \begin{cases} A = -1 \\ B = 2 \\ C = 1 \end{cases}$$

$$\Rightarrow \int \frac{-4x+12}{x^2(x+6)}\,dx = \int \frac{-1x+2}{x^2}\,dx + \int \frac{1}{(x+6)}\,dx =$$

$$= -\int \frac{dx}{x} + 2\int x^{-2}\,dx + \int \frac{dx}{(x+6)} = -\ln|x| - \frac{2}{x} + \ln|x+6| =$$

$$= \ln\left|\frac{x+6}{x}\right| - \frac{2}{x} + C$$

49) $\displaystyle\int \frac{\sqrt{tg\,x + 4}}{\cos^2 x}\,dx =$

$tg\,x + 4 = p$

$\frac{1}{\cos^2 x}\,dx = dp$

$= \int \sqrt{p}\,dp = \int p^{\frac{1}{2}}\,dp = \frac{2}{3}\,p^{\frac{3}{2}} = \frac{2}{3}\sqrt{tg\,x + 4}^{\,3} + C$

50) $\displaystyle\int \sin^2 x \cos^3 x\,dx =$

$\sin^2 x + \cos^2 x = 1$

$\sin 2x = 2\sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$= \int (1 - \cos^2 x)\cos^3 x\,dx = \int (\cos^3 x - \cos^5 x)\,dx =$

$= \underbrace{\int \cos^3 x\,dx}_{I)} - \underbrace{\int \cos^5 x\,dx}_{II)} =$

I) $\displaystyle\int \cos^3 x\,dx =$