AKADEMIA EDUKACYJNO - INŻYNIERSKA

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MATURA STUDIA PRAKTYKA STAŻ PRACA BIZNES PRZEMYSŁ STEAM

KOMPLEKSOWE WSPARCIE EDUKACYJNE **ON – LINE** NA KAŻDYM ETAPIE KSZTAŁCENIA INŻYNIERSKIEGO

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MECHANIKA II

"DYNAMIKA PUNKTU MATERIALNEGO – RUCH HARMONICZNY"

PROJEKT

Oblicz:

- równania przemieszczeń
- równania prędkości
- okres i częstotliwość

punktu materialnego w postaci ogólnych równań ruchu harmonicznego.

DANE:

1) Θ, δ, m_1, F_{W1}
2) $m_2 = OK$
3) $k_3 = OK$
4) $c_4 = OK$
5) $k_5 = OK$

SZUKAM:

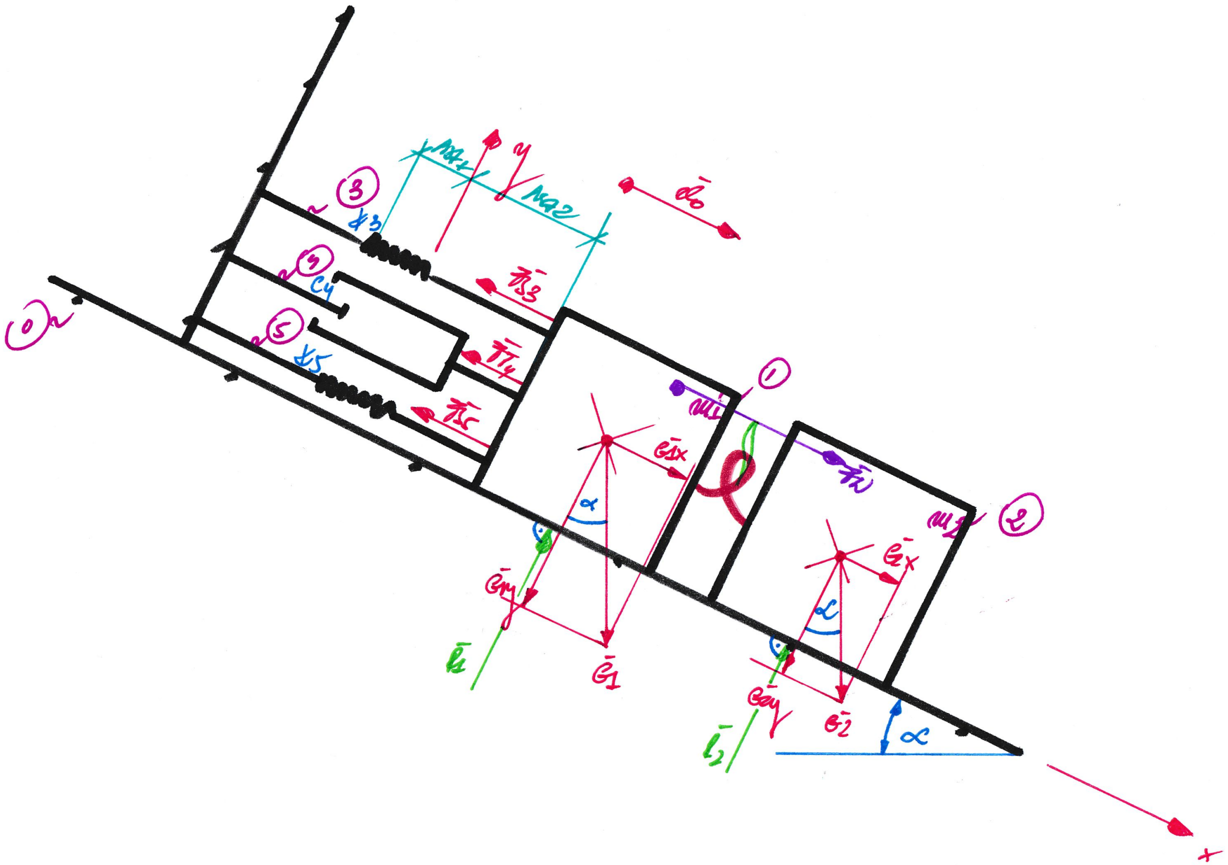
$x_1(t)$, $x_1'(t)$, T, $f = ?$

M2D)

D: $k_3 = k_5 = 10$ [N/m] $\qquad F_W = \theta \sin \vartheta t \qquad \alpha = 30°$ $\qquad x_1(t) = ?$

$m_1 = 2$ [kg] ; $m_2 = \frac{1}{2}$ [kg] $\qquad \theta = 0{,}1$

$c_4 = 3 \qquad \vartheta = 3$



$\bar{F} = m \cdot \bar{a}$

$x(t),\ y(t)$

I) $\sum F_{ix} = m_1\ddot{x} = \bar{G}_{1x} - \bar{F}_{S3} - \bar{F}_{S5} - \bar{F}_{T4} + \bar{F}_N$

II) $\sum F_{iy} = m_1\ddot{y} = \bar{N}_1 - \bar{G}_{y1}$

II) $m_1\overset{\to 0}{\ddot{y}} = \bar{N}_1 - \bar{G}_{y1}$

$N_1 = G_{y1}$

$\frac{G_{1x}}{G_1} = \sin\alpha \Rightarrow G_x = m_1 g \sin\alpha$

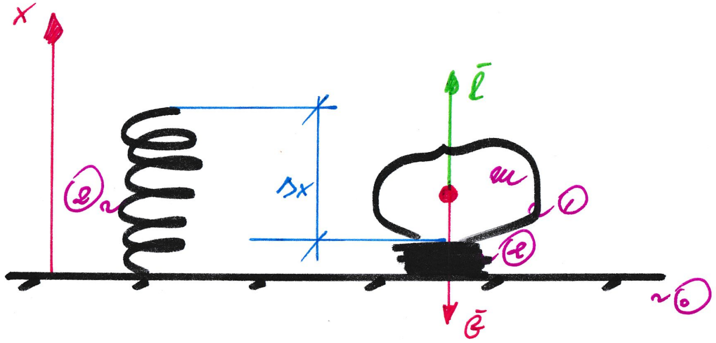
$\frac{G_y}{G} = \cos\alpha \Rightarrow G_y = m_1 g \cos\alpha$

$\Rightarrow N_1 = m_1 g \cos\alpha$

I) $m_1\ddot{x} = \bar{G}_{1x} - \bar{F}_{S3} - \bar{F}_{T4} - \bar{F}_{S5} + \bar{F}_N$

$G_{1x} = m_1 g \sin\alpha$

$F_N = Q\sin\omega t$



$\sum F_{ix} = 0$

$N - G = 0$

$N = G$

$N = m \cdot g$

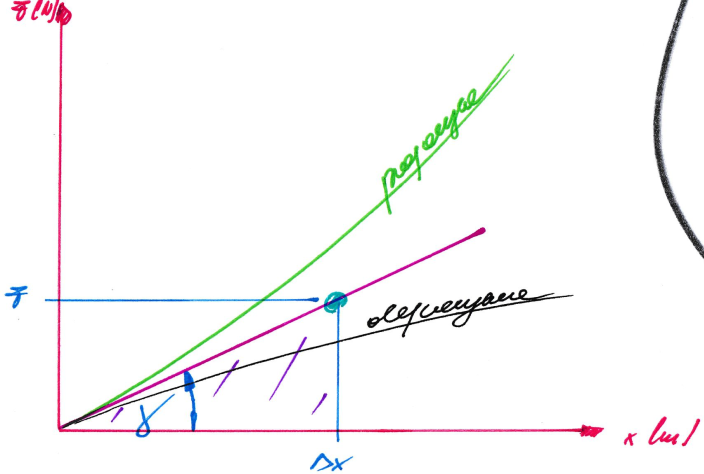
$k \cdot \Delta x = m \cdot g$

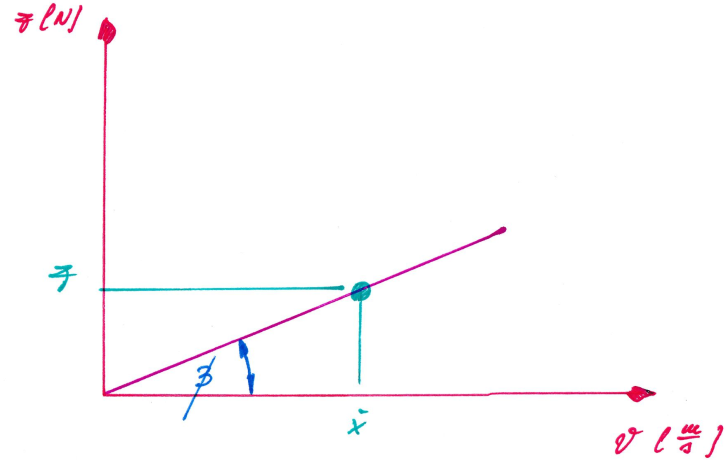
$\Rightarrow F_{S3} = k_3 \cdot \Delta x_3$

$F_{S5} = k_5 \cdot \Delta x_5$

$$\operatorname{tg}\gamma = \frac{F}{\Delta x} \Rightarrow F = \Delta x \cdot \operatorname{tg}\gamma \to k$$

$$F_3 = k \cdot \Delta x$$





$$\operatorname{tg}\beta = \frac{F}{\dot{x}} \Rightarrow F_T = \dot{x} \cdot \operatorname{tg}\beta \to c$$

$$F_T = c \cdot \dot{x}$$

$$N = F \cdot \Delta x$$

$$\Rightarrow N = \frac{1}{2} F \cdot \Delta x = \frac{1}{2} k \Delta x^2$$

$$m_1 \ddot{x}(t) = G_1 x - F_3 - F_{T} - F_5 + F_A$$

$$m_1 \ddot{x} = m g \sin\alpha - k_3 \cdot \Delta x_3 - c \cdot \dot{x} - k_5 \Delta x_5 + Q \sin\Omega t$$

$$m_1 \ddot{x} = m g \sin\alpha - k_3 (x + \Delta st_1) - k_5 (x + \Delta st_1) - c\dot{x} + Q \sin\Omega t$$

$$k_3 = k_5 = k$$

$$m_1 \ddot{x} = m g \sin\alpha - 2k (x + \Delta st_1) - c\dot{x} + Q \sin\Omega t$$

$$\sum F_{ix} = 0 \Rightarrow G_{1x} - F_{e1} = 0$$

$$m_1 g \sin\alpha = +k \cdot \Delta s_{t1}$$

$$\Delta s_{t1} = \frac{m_1 g \sin\alpha}{k}$$

$$m_1 \ddot{x} = m_1 g \sin\alpha - k\left(x + \frac{m_1 g \sin\alpha}{k}\right) - c\dot{x} + Q\sin\omega t$$

$$m_1 \ddot{x} = \cancel{m_1 g \sin\alpha} - kx - \cancel{m_1 g \sin\alpha} - c\dot{x} + Q\sin\omega t$$

$$m_1 \ddot{x} + c\dot{x} + kx = Q\sin\omega t \quad /: m_1$$

$$\ddot{x}(t) + \overset{2n}{\frac{c}{m_1}}\dot{x}(t) + \overset{\omega_0^2}{\frac{k}{m_1}}x(t) = \frac{Q}{m_1}\sin\omega t$$

$$\ddot{x}(t) + 1{,}5\,\dot{x}(t) + 10\,x(t) = 0{,}05\sin 3t$$

$$x(t) = x_j(t) + x_p(t)$$

$$\ddot{x} \to r^2$$
$$\dot{x} \to r^1$$
$$x \to r^0 = 1$$

$x_j(t)$

$$r^2 + 1{,}5r + 10 = 0$$

$$\Delta = b^2 - 4ac$$
$$\Delta = (1{,}5)^2 - 4\cdot 1\cdot 10$$
$$\Delta = -37{,}75 \quad ; \quad i^2 = -1$$
$$\Delta = 37{,}75\, i^2$$
$$\sqrt{\Delta} = \sqrt{37{,}75\, i^2} = 6{,}14i$$

$$\begin{cases} r_1 = \dfrac{-1{,}5 - 6{,}14i}{2} = -0{,}75 - 3{,}07i \\ r_2 = \dfrac{-1{,}5 + 6{,}14i}{2} = -0{,}75 + 3{,}07i \end{cases}$$

$$\begin{cases} \alpha = -0{,}75 \\ \beta = 3{,}07 \end{cases}$$

$$x_j(t) = e^{\alpha t}(C_1 \sin\beta t + C_2\cos\beta t)$$

$$x_j(t) = e^{-0{,}75t}(C_1\sin 3{,}07t + C_2\cos 3{,}07t)$$

$$n = \frac{c}{2m_1} = \frac{3}{2\cdot 2} = 0{,}75$$

$$\omega_0 = \sqrt{\frac{k}{m_1}} = \sqrt{\frac{2\cdot 10}{2}} = 3{,}16$$

$$\omega = \sqrt{\omega_0^2 - n^2}$$

$$\omega = \sqrt{(3{,}16)^2 - (0{,}75)^2} = 3{,}07$$

$x_p(t)$ → $r(t) = 0{,}05 \sin 3t$

$$\begin{cases} x_p(t) = A \sin 3t + B \cos 3t \\ x_p'(t) = 3A \cos 3t - 3B \sin 3t \\ x_p''(t) = -9A \sin 3t - 9B \cos 3t \end{cases}$$

$$\ddot{x}(t) + 1{,}5\dot{x}(t) + 10x(t) = 0{,}05 \sin 3t$$

$$-9A \sin 3t - 9B \cos 3t + 1{,}5(3A \cos 3t - 3B \sin 3t)$$
$$+ 10(-9A \sin 3t - 9B \cos 3t) = 0{,}05 \sin 3t$$

$$\Rightarrow \begin{cases} A = 0{,}0013 \\ B = -0{,}0053 \end{cases}$$

$$x_p(t) = A \sin 3t + B \cos 3t$$

$$\Rightarrow x_p(t) = 0{,}0013 \sin 3t - 0{,}0053 \cos 3t$$

$$x(t) = x_j(t) + x_p(t)$$

$$\Rightarrow {}' \; x(t) = e^{-0{,}45t} \cdot (C_1 \sin 3{,}04t + C_2 \cos 3{,}04t)$$
$$+ 0{,}0013 \sin 3t - 0{,}0053 \cos 3t$$

$$\begin{cases} x_p(t) = (At + B) \sin 3t + (Ct + D) \cos 3t \\ x_p'(t) = \dots \\ x_p''(t) = \dots \end{cases}$$

$${}'' \; x'(t) = -0{,}45 e^{-0{,}45} \cdot (C_1 \sin 3{,}04t + C_2 \cos 3{,}04t) +$$
$$e^{-0{,}45t} \cdot (3{,}04 C_1 \cos 3{,}04t - 3{,}04 C_2 \sin 3{,}04t) +$$
$$+ 0{,}0039 \cos 3t + 0{,}00144 \sin 3t$$

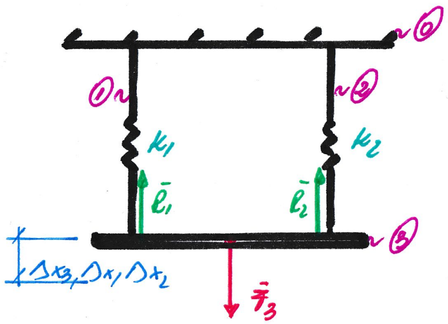
$$[C_2 \cos 3{,}04t]' = C_2 \cdot (\cos 3{,}04t)' = C_2(-\sin 3{,}04t) \cdot (3{,}04t)' =$$
$$= -3{,}04 C_2 \sin 3{,}04t$$

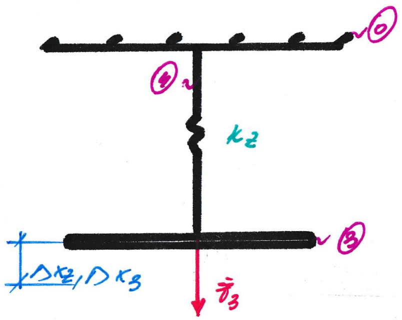
$$\begin{cases} C_1 = 0{,}1289 \\ C_2 = 0{,}03 \\ x(t) = \dots \end{cases}$$

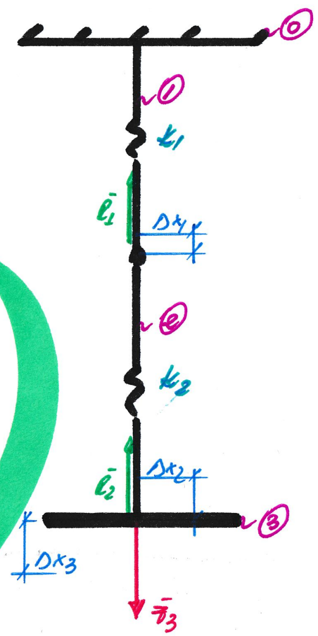
$$\Rightarrow \begin{cases} {}' \; x(t) = \dots \\ {}'' \; x'(t) = \dots \end{cases} \Rightarrow \begin{cases} x(t=0) = A_{12} = \dfrac{m_2 g \sin\alpha}{2k} \\ x'(t=0) = v_1 \end{cases}$$

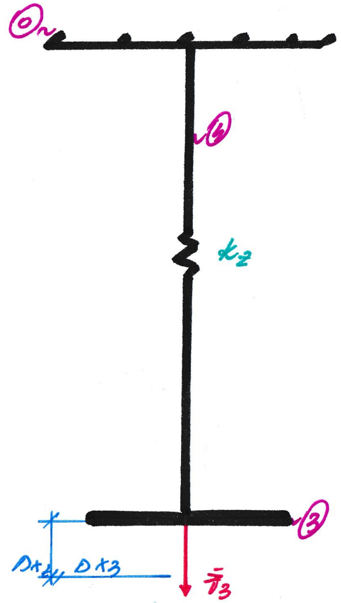
$$\begin{cases} 1) \; \bar{F} = m \cdot \bar{a} \\ 2) \; U = I \cdot R \end{cases}$$

$$\bar{F} = k \cdot \Delta x$$









$$\begin{cases} \bar{F}_1 = k_1 \cdot \Delta x_1 \\ \bar{F}_2 = k_2 \cdot \Delta x_2 \\ \bar{F}_e = k_e \cdot \Delta x_e \end{cases}$$

$$\begin{cases} \bar{F}_1 = k_1 \cdot \Delta x_1 \\ \bar{F}_2 = k_2 \cdot \Delta x_2 \\ \bar{F}_e = k_e \cdot \Delta x_e \end{cases}$$

$$\bar{F}_e = \bar{F}_1 + \bar{F}_2$$

$$k_e \cancel{\Delta x_e} = k_1 \cancel{\Delta x_1} + k_2 \cancel{\Delta x_2} \quad ; \quad \Delta x_e = \Delta x_1 = \Delta x_2$$

$$k_e = k_1 + k_2$$

$$\Delta x_e = \Delta x_1 + \Delta x_2$$

$$\frac{\cancel{\bar{F}_e}}{k_e} = \frac{\cancel{\bar{F}_1}}{k_1} + \frac{\cancel{\bar{F}_2}}{k_2} \quad ; \quad \bar{F}_e = \bar{F}_1 = \bar{F}_2$$

$$\frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2}$$

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