AKADEMIA EDUKACYJNO - INŻYNIERSKA

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MATURA STUDIA PRAKTYKA STAŻ PRACA BIZNES PRZEMYSŁ STEAM

KOMPLEKSOWE WSPARCIE EDUKACYJNE **ON – LINE** NA KAŻDYM ETAPIE KSZTAŁCENIA INŻYNIERSKIEGO

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MECHANIKA II

"DYNAMIKA PUNKTU MATERIALNEGO – RUCH HARMONICZNY"

PROJEKT

Oblicz:

- równania przemieszczeń
- równania prędkości
- okres i częstotliwość

punktu materialnego układu płaskiego w postaci ogólnych równań ruchu harmonicznego korzystając z równań klasycznych, równań La Grange'a, zasady zachowania energii mechanicznej oraz zasady zachowania pędu (krętu).

DANE:

$l, s_0, V_0, \varphi_0, \omega_0, m = OK$

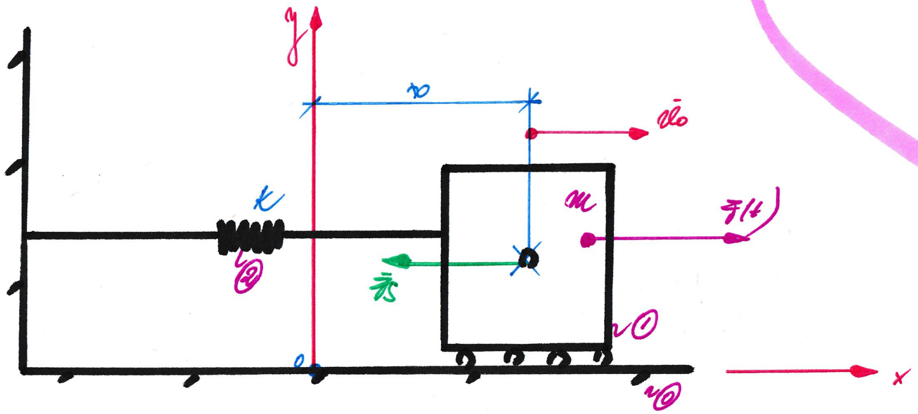
SZUKAM:

$x(t), x'(t), \varphi(t), \varphi'(t)\ T, f = ?$

1)

$m, k = \checkmark$ $\qquad x(t) = ?$

$\bar{F} = m \cdot \bar{a}$



$\Sigma F_x = m\ddot{x} = -F_s$

$m\ddot{x} = -kx$

$m\ddot{x} + kx = 0 \quad /: m$

$\ddot{x}(t) + \underbrace{\frac{k}{m}}_{\omega_0^2} x(t) = 0$

$\ddot{x} \to r^2$

$\dot{x} \to r^1 = r$

$x \to r^0 = 1$

$r^2 + \omega_0^2 = 0$

$r^2 = -\omega_0^2 \quad ; \quad i^2 = -1$

$r^2 = i^2\omega_0^2$

$\begin{cases} r_1 = i\omega_0 \\ r_2 = -i\omega_0 \end{cases} \Rightarrow \begin{cases} \alpha = 0 \\ \beta = \omega_0 \end{cases}$

$x(t) = e^{\alpha t}(C_1 \sin\beta t + C_2 \cos\beta t)$

$x(t) = e^{0}(C_1 \sin\omega_0 t + C_2 \cos\omega_0 t)$

1) $x(t) = C_1 \sin\omega_0 t + C_2 \cos\omega_0 t$

2) $\dot{x}(t) = \omega_0 C_1 \cos\omega_0 t - \omega_0 C_2 \sin\omega_0 t$

1) $x(t=0) = x_0$

2) $\dot{x}(t=0) = v_0$

1) $x_0 = C_1 \cancel{\sin(0)} + C_2 \cos(0)$

2) $v_0 = \omega_0 C_1 \cos(0) - \omega_0 C_2 \cancel{\sin(0)}$

1) $C_1 = \frac{v_0}{\omega_0}$

2) $C_2 = x_0$

1) $x(t) = \frac{v_0}{\omega_0} \sin\omega_0 t + x_0 \cos\omega_0 t$

2) $\dot{x}(t) = v_0 \cos\omega_0 t - \omega_0 x_0 \sin\omega_0 t$

1) $x(t) = x_0 \cos\omega_0 t$

2) $\dot{x}(t) = -\omega_0 x_0 \sin\omega_0 t$

T = TIME of MOVE

1) $x(t=T) = x_K$

2) $\dot{x}(t=T) = v_K = 0$

1) $x_K = x_0 \cos\omega_0 T$

2) $0 = -\omega_0 x_0 \sin\omega_0 T$??

'') $0 = -\omega_0 x_0 \sin \omega_0 T$

$0 = \sin \omega_0 T$

$\omega_0 T = 0 + 2k\pi \quad ; \quad k = 1$

$\omega_0 T = 2\pi$

$T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{m}{k}}$

$f = \frac{1}{T} \rightarrow f = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

') $x_K = x_0 \cos \omega_0 T$

$x_K = x_0 \cos \cancel{\omega_0} \cdot \frac{2\pi}{\cancel{\omega_0}}$

$x_K = x_0$

τ = TIME of MOVE

') $x(t=\tau) = x_K = 0$

'') $\dot{x}(t=\tau) = v_K$

') $0 = x_0 \cos \omega_0 \tau$

'') $v_K = -\omega_0 x_0 \sin \omega_0 \tau$??

') $0 = x_0 \cos \omega_0 \tau \quad /: x_0$

$\cos \omega_0 \tau = 0$

$\omega_0 \tau = \frac{\pi}{2} + 2k\pi$ ^{0}

$\tau = \frac{\pi}{2\omega_0}$

'') $v_K = -\omega_0 x_0 \sin \omega_0 \tau$

$v_K = -\omega_0 x_0 \sin \cancel{\omega_0} \frac{\pi}{2\cancel{\omega_0}}$

$v_K = -\omega_0 x_0$

$E_{TOT} = E_K + E_P \qquad q = x$

$E_{TOT} = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} k x^2$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = Q(t)$

$L = E_K - E_P \qquad q = x$

$L = \frac{1}{2} m \dot{x}^2 - \frac{1}{2} k x^2$

$\frac{\partial L}{\partial x} = -kx$

$\frac{\partial L}{\partial \dot{x}} = m\dot{x}$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = m\ddot{x}$

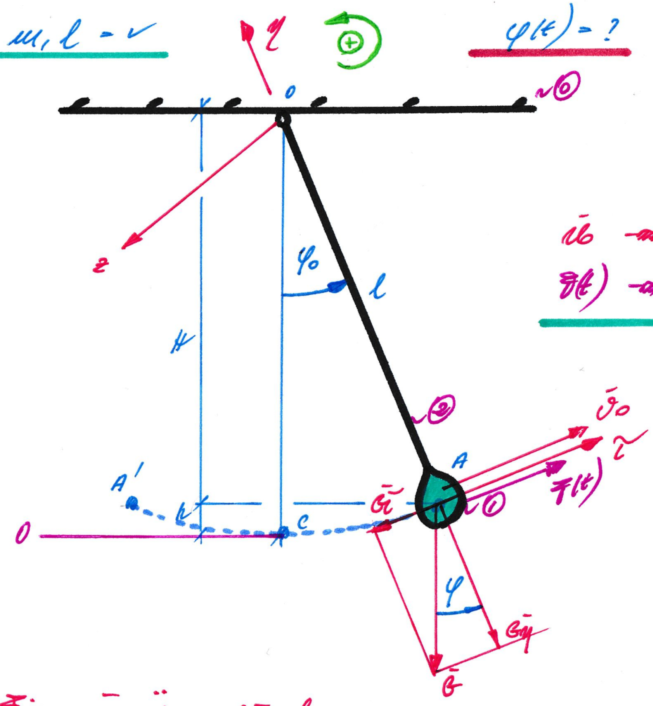
$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$

$m\ddot{x} - (-kx) = 0$

$m\ddot{x} + kx = 0 \quad /: m$

$\ddot{x}(t) + \frac{k}{m} x(t) = 0$

$m, l = v$ $\qquad$ $\varphi(t) = ?$



$\vartheta_0 \Rightarrow \omega_0$

$F(t) \Rightarrow M(t)$

$\sum M_{iz} = I_z\ddot{\varphi} = -G_\tau \cdot l$

$\frac{G_\tau}{G} = \sin\varphi \Rightarrow G_\tau = mg\sin\varphi$

$\frac{G_y}{G} = \cos\varphi \Rightarrow G_y = mg\cos\alpha$

$I = ml^2$

$\Rightarrow ml^2\ddot{\varphi} + mgl\overset{\varphi}{\sin\varphi} = 0 \quad /: ml^2$

$\ddot{\varphi}(t) + \overset{\omega_0^2}{\frac{g}{l}}\varphi(t) = 0$

$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{\cancel{kg}\cdot \cancel{m}/s^2}{\cancel{m}}\cdot\frac{1}{\cancel{kg}}} = \left[\frac{1}{s}\right]$

$\omega_0 = \sqrt{\frac{g}{l}} = \sqrt{\frac{\cancel{m}/s^2}{\cancel{m}}} = \left[\frac{1}{s}\right]$

$\frac{d}{dt}\left(\frac{dL}{d\dot{q}}\right) - \frac{dL}{dq} = Q(t)$ $\qquad$ $q = \varphi$

$L = E_k - E_p$

$E_k = \frac{1}{2} I_z \cdot \dot{\varphi}^2$ $\qquad$ $\frac{H}{l} = \cos\varphi \Rightarrow H = l\cos\varphi$

$E_p = mgh = mgl(1-\cos\varphi)$ $\qquad$ $h = l - H = l(1-\cos\varphi)$

$\Rightarrow L = \frac{1}{2} I_z\dot{\varphi}^2 + mgl\cos\varphi - mgl$

$\frac{d}{dt}\left(\frac{dL}{d\dot{\varphi}}\right) - \frac{dL}{d\varphi} = 0$

$\frac{dL}{d\varphi} = -mgl\sin\varphi$

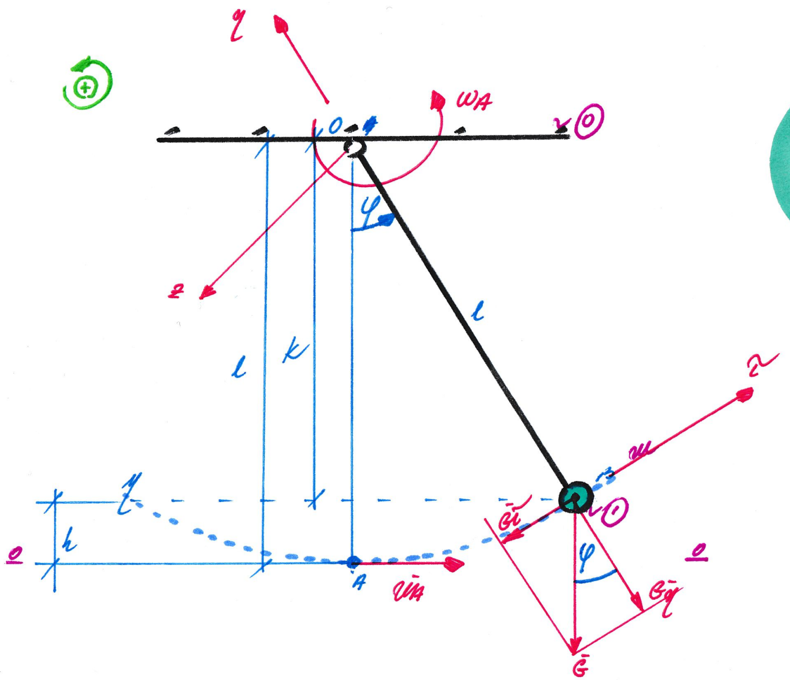
$\frac{dL}{d\dot{\varphi}} = ml^2\dot{\varphi}$

$\frac{d}{dt}\left(\frac{dL}{d\dot{\varphi}}\right) = ml^2\ddot{\varphi}$

$ml^2\ddot{\varphi} - (-mgl\overset{\varphi}{\sin\varphi}) = 0$

$\ddot{\varphi}(t) + \frac{g}{l}\varphi(t) = 0$

81) D: $m, l, v_A = \checkmark$ sz: $h, \theta = ?$



$$\begin{cases} \bar{F} = m \cdot \bar{a} \\ \bar{M} = J \cdot \bar{\varepsilon} \end{cases}$$

$$\sum M_{iz} = J_z^{(0)} \ddot{\varphi} = -G_t \cdot l$$

$$\frac{G_n}{G} = \cos\varphi \Rightarrow G_n = mg\cos\varphi$$

$$\frac{G_t}{G} = \sin\varphi \Rightarrow G_t = mg\sin\varphi$$

$$ml^2\ddot{\varphi} = -mgl\sin\varphi$$

$$ml^2\ddot{\varphi} + mgl\sin\varphi = 0 \quad /: ml^2$$

$\sin\varphi \approx \varphi$

$\cos\varphi \approx 1$

$$\ddot{\varphi}(t) + \underbrace{\frac{g}{l}}_{\omega_0^2}\varphi(t) = 0$$

$\Rightarrow \varphi(t) = \dots$

$$\frac{k}{l} = \cos\varphi$$

$k = l\cos\varphi$

$l - h = l\cos\varphi$

$h = l - l\cos\varphi$

$h = l(1 - \cos\varphi)$

$$\Delta E_k = -\Delta E_p$$

$$\underbrace{\frac{1}{2} I^{(0)} \omega_B^2}_{0} - \frac{1}{2} I^{(0)} \omega_A^2 = -mgh$$

$$\Rightarrow \quad h = \frac{I^{(0)}\omega_A^2}{2gm} = \frac{\cancel{m}\cancel{l^2} \cdot \frac{v_m^2}{\cancel{l^2}}}{2g\cancel{m}} = \frac{v_m^2}{2g}$$

$$\Rightarrow \begin{cases} (') \; h = l(1-\cos\varphi) \\ ('') \; h = \dfrac{v_m^2}{2g} \end{cases}$$

$('\,,\,'')$

$$l(1-\cos\varphi) = \frac{v_m^2}{2g}$$

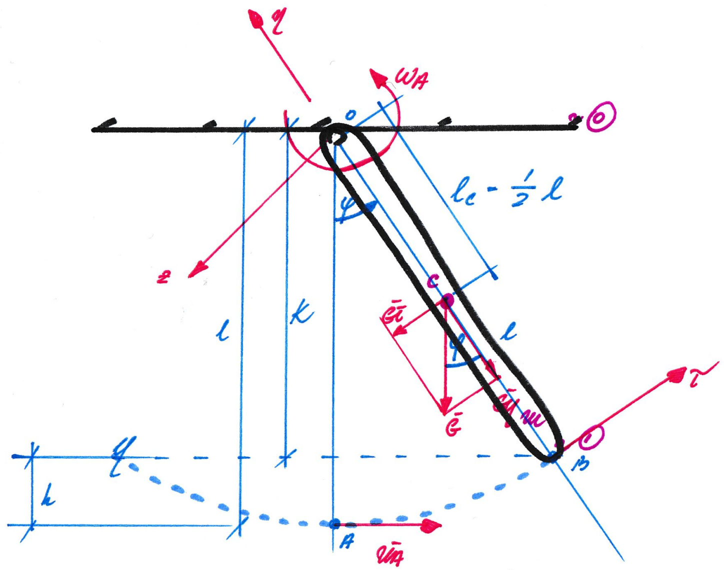
$$1-\cos\varphi = \frac{v_m^2}{2gl}$$

$$\cos\varphi = 1 - \frac{v_m^2}{2gl}$$

$$\varphi = \arccos\left(1 - \frac{v_m^2}{2gl}\right)$$

$$\begin{cases} \ddot{\varphi}(t) + \dfrac{g}{l}\varphi(t) = 0 \\ \varphi = \arccos\left(1 - \dfrac{v_m^2}{2gl}\right) \end{cases}$$

82) D: $m, l, v_A = \checkmark$ Sz: $h, \theta = ?$



$$\begin{cases} \bar{F} = m \cdot \bar{a} \\ \bar{M} = \bar{I} \cdot \bar{\varepsilon} \end{cases}$$

$$\sum M_{iz} = I^{(0)} \ddot{\varphi} = - G_\tau \cdot l_c$$

$$\frac{G_n}{G} = \cos\varphi \Rightarrow G_n = mg\cos\varphi$$

$$\frac{G_\tau}{G} = \sin\varphi \Rightarrow G_\tau = mg\sin\varphi$$

$$\frac{1}{3} m l^2 \ddot{\varphi} = - mg \underbrace{\sin\varphi}_{\varphi} \cdot \frac{1}{2} l$$

$$\frac{1}{3} m l^2 \ddot{\varphi} + \frac{1}{2} mgl\varphi = 0 \quad / \cdot \frac{3}{ml^2}$$

$$\ddot{\varphi}(t) + \overbrace{\frac{3g}{2l}}^{\omega_0^2} \varphi(t) = 0$$

$\Rightarrow \varphi(t) = \ldots$

$$\begin{cases} \ddot{\varphi}(t) + \frac{3g}{2l}\varphi(t) = 0 \\ \varphi = \arccos\left(1 - \frac{v_A^2}{3gl}\right) \end{cases}$$

$$\frac{k}{l} = \cos\varphi$$

$$k = l\cos\varphi$$

$$l - h = l\cos\varphi$$

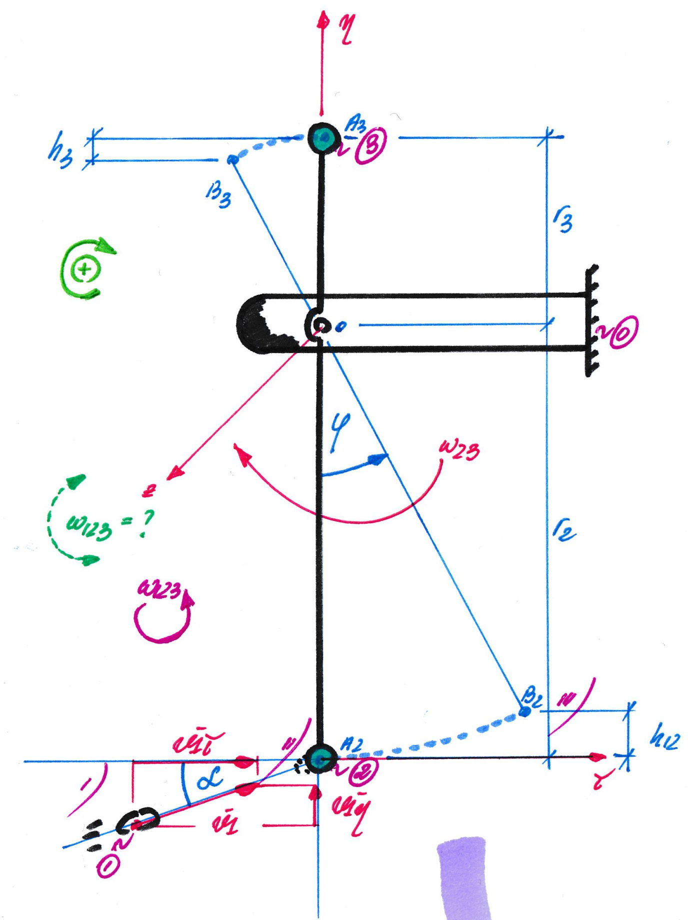
$$h = l(1 - \cos\varphi)$$

$$\Delta E_k = - \Delta E_p$$

$$\underbrace{\frac{1}{2} I^{(0)} \omega_B^2}_{0} - \frac{1}{2} I^{(0)} \omega_A^2 = - mgh$$

$$\Rightarrow h = \frac{I^{(0)} \omega_A^2}{2mg} = \frac{\frac{1}{3} \cancel{m} \cancel{l^2} \cdot \frac{v_A^2}{\cancel{l^2}}}{2\cancel{m}g} = \frac{v_A^2}{6g}$$

33)



$D: m_2 = m_3 = 3{,}2 \ (kg)$ $\quad m_1 = 0{,}05 \ (kg)$

$\omega_{23} = 6 \ (rad/s)$ $\quad v_1 = 300 \ (m/s)$

$r_2 = 0{,}4 \ (m)$

$r_3 = 0{,}2 \ (m)$

$\alpha = 20°$

I) $\Delta K = K_{AFT} - K_{BEF}$

II) $\Delta E_k = E_{k\,AFT} - E_{k\,BEF}$

III) $\Delta E_k = \pm \Delta E_p$

I) $\overset{0}{\Delta K} = K_{AFT} - K_{BEF}$

I, II)

$$K_{AFT} = \bar{I}_2^{(0)} \omega_{123} + \bar{I}_3^{(0)} \omega_{123} + \bar{I}_1^{(0)} \omega_{123}$$

$$K_{BEF} = \bar{I}_2^{(0)} \omega_{23} + \bar{I}_3^{(0)} \omega_{23} - \bar{I}_1^{(0)} \omega_1$$

$$\bar{I}_1^{(0)} = m_1 r_2^2 \qquad \frac{v_{1\tau}}{v_1} = \cos\alpha \Rightarrow v_{1\tau} = v_1 \cos\alpha$$

$$\bar{I}_2^{(0)} = m_2 r_2^2 \qquad \omega_1 = \frac{v_{1\tau}}{r_2} = \frac{v_1 \cos\alpha}{r_2}$$

$$\bar{I}_3^{(0)} = m_3 r_3^2$$

$$\Rightarrow 0 = \left(\bar{I}_2^{(0)} + \bar{I}_3^{(0)} + \bar{I}_1^{(0)}\right)\omega_{123} - \left(\bar{I}_2^{(0)}\omega_{23} + \bar{I}_3^{(0)}\omega_{23} - \bar{I}_1^{(0)}\omega_1\right)$$

$$\Rightarrow \omega_{123} = \frac{\bar{I}_2^{(0)}\omega_{23} + \bar{I}_3^{(0)}\omega_{23} - \bar{I}_1^{(0)}\omega_1}{\bar{I}_2^{(0)} + \bar{I}_3^{(0)} + \bar{I}_1^{(0)}} = -2{,}46 \ (rad/s)$$

$$= \frac{m_2 r_2^2 \omega_{23} + m_3 r_3^2 \omega_{23} - m_1 r_2^{\cancel{2}} \frac{v_1 \cos\alpha}{\cancel{r_2}}}{m_2 r_2^2 + m_3 r_3^2 + m_1 r_2^2} = m_2 = m_3 = m_{23} = \frac{m_{23}(r_2^2 + r_3^2)\omega_{23} - m_1 r_2 v_1 \cos\alpha}{m_{23}(r_2^2 + r_3^2) + m_1 r_2^2} = \frac{3{,}2(0{,}4^2 + 0{,}2^2)6 - 0{,}05 \cdot 0{,}4 \cdot 300 \cos 20°}{3{,}2(0{,}4^2 + 0{,}2^2) + 0{,}05 \cdot (0{,}4)^2} = \frac{-1{,}45}{0{,}648}$$

12)

$$\Delta E_k = -\Delta E_p$$

$$\underbrace{\cancel{\frac{1}{2}\bar{I}_{12}^{(0)}\omega_{B_2}^{2}}}_{0} - \frac{1}{2}\bar{I}_{12}^{(0)}\omega_{A_2}^{2} = -m_{12}\,g\,h_{12}$$

$$\Rightarrow h_{12} = \frac{\bar{I}_{12}^{(0)}\omega_{A_2}^{2}}{2m_{12}g} = \frac{(m_1r_2^2+m_2r_2^2)\,\omega_{123}^2}{2(m_1+m_2)g}$$

$$\Rightarrow h_{12} = r_2 - r_2\cos\varphi = r_2(1-\cos\varphi)$$

3)

$$\Delta E_k = \Delta E_p$$

$$\underbrace{\cancel{\frac{1}{2}\bar{I}_{3}^{(0)}\omega_{B_3}^{2}}}_{0} - \frac{1}{2}\bar{I}_{3}^{(0)}\omega_{A_3}^{2} = m_3\,g\,h_3$$

$$\Rightarrow h_3 = \frac{-\bar{I}_{3}^{(0)}\omega_{A_3}^{2}}{2m_3g} = \frac{-m_3r_3^2\cdot\omega_{123}^2}{2m_3g}$$

$$\Rightarrow h_3 = r_3 - r_3\cos\varphi = r_3(1-\cos\varphi)$$

123)

$$\Delta E_{k123} = -\Delta E_{p123}$$

$$-\frac{1}{2}\bar{I}_{12}^{(0)}\omega_{A_2}^{2} - \frac{1}{2}\bar{I}_{3}^{(0)}\omega_{A_3}^{2} = -m_{12}\,g\,h_{12} + m_3\,g\,h_3$$

$$-\frac{(m_1r_2^2+m_2r_2^2)\,\omega_{123}^2}{2} - \frac{m_3r_3^2\cdot\omega_{123}^2}{2} = m_3gr_3(1-\cos\varphi) - (m_1+m_2)gr_2(1-\cos\varphi)$$

$$(m_1r_2^2+m_2r_2^2+m_3r_3^2)\,\omega_{123}^2 = 2(m_1+m_2)gr_2(1-\cos\varphi) - 2m_3gr_3(1-\cos\varphi)$$

$$(m_1r_2^2+m_2r_2^2+m_3r_3^2)\,\omega_{123}^2 = 2g\left[(m_1+m_2)r_2 - m_3r_3\right](1-\cos\varphi)$$

$$1-\cos\varphi = \frac{(m_1r_2^2+m_2r_2^2+m_3r_3^2)\,\omega_{123}^2}{2g\left[(m_1+m_2)r_2 - m_3r_3\right]}$$

$$\varphi = \operatorname{acos}\left[1 - \frac{(m_1r_2^2+m_2r_2^2+m_3r_3^2)\,\omega_{123}^2}{2g\left[(m_1+m_2)r_2 - m_3r_3\right]}\right]$$

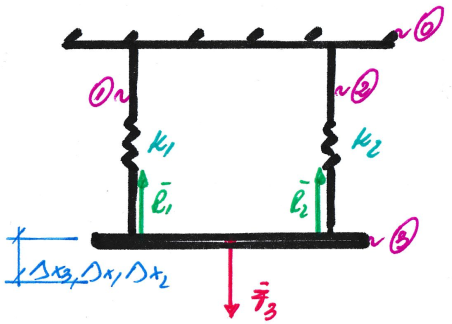
$$\Rightarrow \varphi = \operatorname{acos}\left[1 - \frac{\left[(0{,}05)(0{,}4)^2+(3{,}2)(0{,}4)^2+(3{,}2)(0{,}2)^2\right](\text{[illegible]})^2}{2\cdot 9{,}81\left[(0{,}05+3{,}2)(0{,}4)-(3{,}2)(0{,}2)\right]}\right] =$$

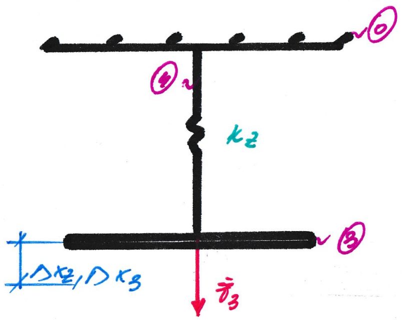
$$= \operatorname{acos}\left[1 - \frac{4{,}83}{12{,}89}\right] =$$

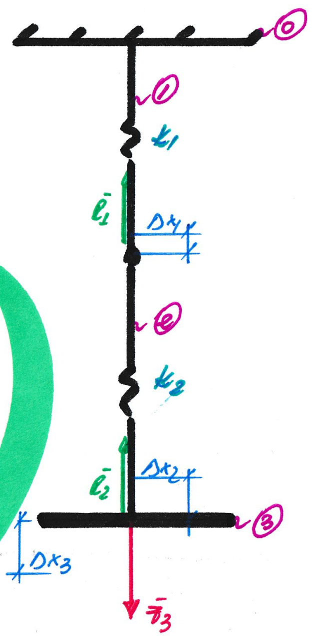
$$= \operatorname{acos}(0{,}6181) \simeq 52^{\circ}$$

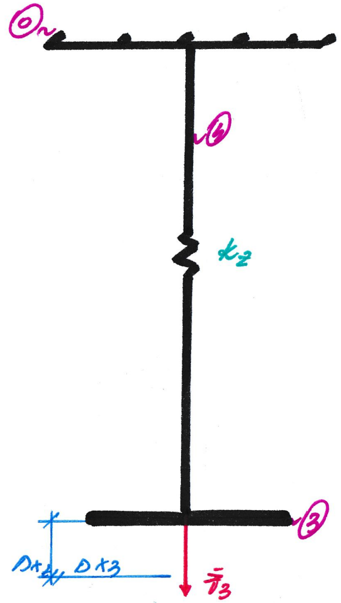
$$\begin{cases} \bar{F} = m \cdot \bar{a} \\ U = \bar{I} \cdot R \end{cases}$$

$$\bar{F} = k \cdot \Delta x$$









$$\begin{cases} \bar{F}_1 = k_1 \cdot \Delta x_1 \\ \bar{F}_2 = k_2 \cdot \Delta x_2 \\ \bar{F}_e = k_e \cdot \Delta x_e \end{cases}$$

$$\begin{cases} \bar{F}_1 = k_1 \cdot \Delta x_1 \\ \bar{F}_2 = k_2 \cdot \Delta x_2 \\ \bar{F}_e = k_e \cdot \Delta x_e \end{cases}$$

$$\bar{F}_e = \bar{F}_1 + \bar{F}_2$$

$$k_e \cancel{\Delta x_e} = k_1 \cancel{\Delta x_1} + k_2 \cancel{\Delta x_2} \quad ; \quad \Delta x_e = \Delta x_1 = \Delta x_2$$

$$k_e = k_1 + k_2$$

$$\Delta x_e = \Delta x_1 + \Delta x_2$$

$$\frac{\cancel{\bar{F}_e}}{k_e} = \frac{\cancel{\bar{F}_1}}{k_1} + \frac{\cancel{\bar{F}_2}}{k_2} \quad ; \quad \bar{F}_e = \bar{F}_1 = \bar{F}_2$$

$$\frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2}$$