AKADEMIA EDUKACYJNO - INŻYNIERSKA

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MATURA STUDIA PRAKTYKA STAŻ PRACA BIZNES PRZEMYSŁ STEAM

KOMPLEKSOWE WSPARCIE EDUKACYJNE **ON – LINE** NA KAŻDYM ETAPIE KSZTAŁCENIA INŻYNIERSKIEGO

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MECHANIKA II

"DYNAMIKA PUNKTU MATERIALNEGO – RUCH HARMONICZNY"

PROJEKT

Oblicz:

- równania przemieszczeń
- równania prędkości
- okres i częstotliwość

punktu materialnego w postaci ogólnych równań ruchu harmonicznego.

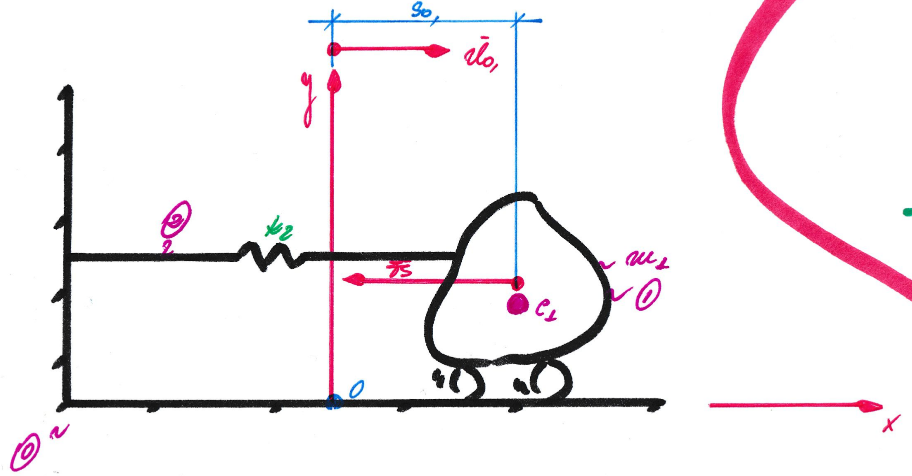
DANE:

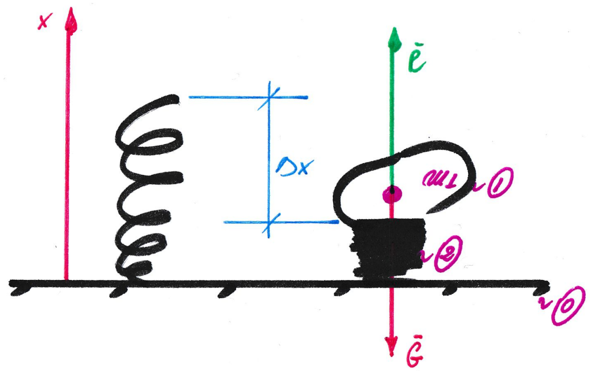
1) $m_1, s_{01}, V_{01} = OK$
2) $k_2 = OK$

SZUKAM:

$x(t), x'(t), T, f = ?$

1) m_1, k_2, s_0, v_0 $x(t) = ?$





$\sum F_{ix} = 0$

$\vec{E} - \vec{G} = 0$

$E = m \cdot g$

$k \cdot \Delta x = m \cdot g$

$\Rightarrow k = \frac{mg}{\Delta x}$

$\vec{F} = m \cdot \vec{a} \Rightarrow \sum F_{ix} = m\ddot{x} = -F_s$

$m\ddot{x} = -kx$

$m\ddot{x} + kx = 0 \ /: m$

$\ddot{x}(t) + \overbrace{\frac{k}{m}}^{\omega_0^2} x(t) = 0$

$\ddot{x} = r^2$

$\dot{x} = r$

$x = r^0 = 1$

$\Rightarrow r^2 + \omega_0^2 = 0$

$r^2 = -\omega_0^2$ $i^2 = -1$

$r^2 = i^2 \omega_0^2$

$r = \pm i\omega_0$

$r = 0 \pm i\omega_0 \Rightarrow \begin{cases} \alpha = 0 \\ \beta = \omega_0 \end{cases}$

$x(t) = e^{\alpha t}(C_1 \sin\beta t + C_2 \cos\beta t) \Rightarrow x(t) = C_1 \sin\omega_0 t + C_2 \cos\omega_0 t$

$$\begin{cases} x(t) = C_1\,\text{ημ}\,\omega_0 t + C_2\,\text{συν}\,\omega_0 t \\ \dot{x}(t) = C_1\omega_0\,\text{συν}\,\omega_0 t - C_2\omega_0\,\text{ημ}\,\omega_0 t \end{cases}$$

$$\begin{cases} x(t=0) = x_{01} \\ \dot{x}(t=0) = \upsilon_{01} \end{cases}$$

$$\begin{cases} x_{01} = C_1\,\cancel{\text{ημ}}(0) + C_2\,\text{συν}(0) \\ \upsilon_{01} = C_1\omega_0\,\text{συν}(0) - C_2\omega_0\,\cancel{\text{ημ}}(0) \end{cases}$$

$$\begin{cases} C_1 = \dfrac{\upsilon_{01}}{\omega_0} \\ C_2 = x_{01} \end{cases}$$

$$\begin{cases} x(t) = \dfrac{\cancel{\upsilon_{01}}^{\,0}}{\omega_0}\,\text{ημ}\,\omega_0 t + x_{01}\,\text{συν}\,\omega_0 t \\ \dot{x}(t) = \cancel{\upsilon_{01}}^{\,0}\,\text{συν}\,\omega_0 t - x_{01}\omega_0\,\text{ημ}\,\omega_0 t \end{cases}$$

$$\begin{cases} x(t) = x_{01}\,\text{συν}\,\omega_0 t = x_{01}\,\text{συν}\,\omega_0 t \\ \dot{x}(t) = -x_{01}\omega_0\,\text{ημ}\,\omega_0 t = -x_{01}\omega_0\,\text{ημ}\,\omega_0 t \end{cases}$$

$T =$ [illegible] (1 [illegible])

$$\begin{cases} x(t=T) = x_{K1} \\ \dot{x}(t=T) = \upsilon_{K1} \end{cases}$$

$$\begin{cases} x_{K1} = x_{01}\,\text{συν}\,\omega_0 T \\ \upsilon_{K1} = -x_{01}\omega_0\,\text{ημ}\,\omega_0 T \end{cases}$$

$$x_{K1} = x_{01}\,\text{συν}\,\omega_0 T$$

$$\cancel{x_{01}} = \cancel{x_{01}}\,\text{συν}\,\omega_0 T$$

$$\begin{cases} \text{συν}\,\omega_0 T = 1 \\ \text{συν}\,2\pi = 1 \end{cases}$$

$$\Rightarrow \quad \omega_0 T = 2\pi$$

$$T = \frac{2\pi}{\omega_0}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$\upsilon_{K1} = -x_{01}\omega_0\,\text{ημ}\,\omega_0 T$$

$$0 = -x_{01}\omega_0\,\text{ημ}\,\omega_0 T$$

$$\begin{cases} \text{ημ}\,\omega_0 T = 0 \\ \text{ημ}\,2\pi = 0 \end{cases}$$

$$\Rightarrow \quad \omega_0 T = 2\pi$$

$$T = \frac{2\pi}{\omega_0}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$f = \frac{1}{T}$$

$$1)\ x(t) = x_{01} \cos \omega_0 t$$
$$2)\ x'(t) = -x_{01} \omega_0 \sin \omega_0 t$$

$\tilde{\tau}$ - [illegible]

$$1)\ x(t = \tilde{\tau}) = x_K = 0$$
$$2)\ \dot{x}(t = \tilde{\tau}) = v_K$$

$$1)\ 0 = x_{01} \cos \omega_0 \tilde{\tau}$$
$$2)\ v_K = -x_{01} \omega_0 \sin \omega_0 \tilde{\tau}$$

$$1)\ 0 = x_{01} \cos \omega_0 \tilde{\tau} \quad / : x_{01}$$

$$\cos \omega_0 \tilde{\tau} = 0$$
$$\omega_0 \tilde{\tau} = \frac{\tilde{\pi}}{2}$$
$$\tilde{\tau} = \frac{\tilde{\pi}}{2\omega_0}$$

$$2)\ v_K = -x_{01} \omega_0 \sin \omega_0 \tilde{\tau}$$

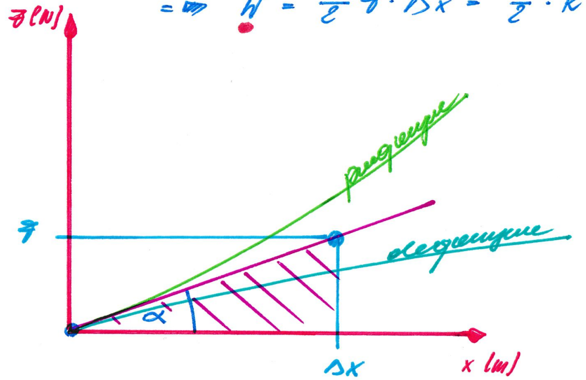
$$v_K = -x_{01} \omega_0 \sin \cancel{\omega_0} \cdot \frac{\tilde{\pi}}{2\cancel{\omega_0}}$$

$$v_K = -x_0 \omega_0 \underbrace{\sin \frac{\tilde{\pi}}{2}}_{1}$$

$$v_K = -x_0 \omega_0 = -x_0 \cdot \sqrt{\frac{k}{m}}$$

$$W = F \cdot s$$

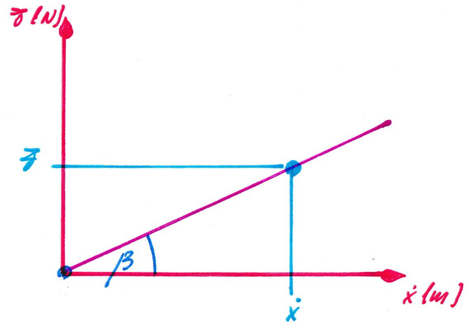
$$\Rightarrow W = \frac{1}{2} F \cdot \Delta x = \frac{1}{2} \cdot k \cdot \Delta x \cdot \Delta x = \frac{1}{2} k \Delta x^2 = \frac{1}{2} k x^2$$



$$\operatorname{tg} \alpha = \frac{F}{\Delta x}$$

$$\Rightarrow \operatorname{tg} \alpha = \frac{F}{\Delta x} = k$$

$$\Rightarrow F = k \cdot \Delta x$$



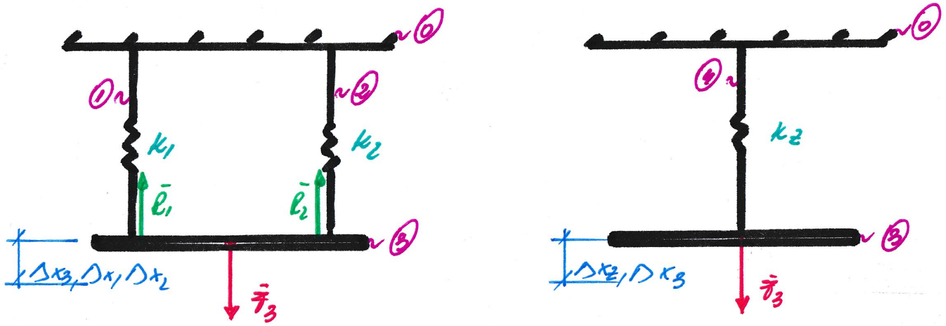
$$\operatorname{tg} \beta = \frac{F}{\dot{x}}$$

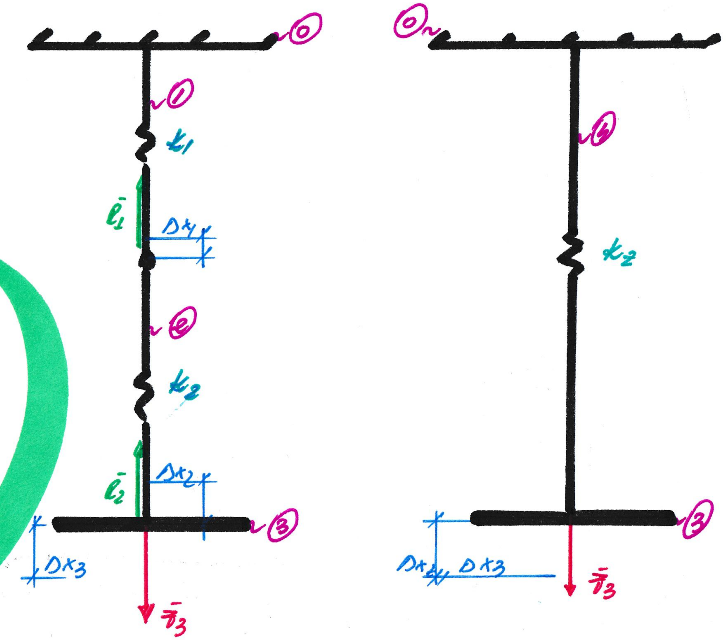
$$\Rightarrow \operatorname{tg} \beta = \frac{F}{\dot{x}} = c$$

$$\Rightarrow F = c \cdot \dot{x}$$

$$\begin{cases} \bar{F} = m \cdot \bar{a} \\ U = \bar{I} \cdot R \end{cases}$$

$$\bar{F} = k \cdot \Delta x$$





$$\begin{cases} \bar{F}_1 = k_1 \cdot \Delta x_1 \\ \bar{F}_2 = k_2 \cdot \Delta x_2 \\ \bar{F}_e = k_e \cdot \Delta x_e \end{cases}$$

$$\begin{cases} \bar{F}_1 = k_1 \cdot \Delta x_1 \\ \bar{F}_2 = k_2 \cdot \Delta x_2 \\ \bar{F}_e = k_e \cdot \Delta x_e \end{cases}$$

$$\bar{F}_e = \bar{F}_1 + \bar{F}_2$$

$$k_e \cancel{\Delta x_e} = k_1 \cancel{\Delta x_1} + k_2 \cancel{\Delta x_2} \quad ; \quad \Delta x_e = \Delta x_1 = \Delta x_2$$

$$k_e = k_1 + k_2$$

$$\Delta x_e = \Delta x_1 + \Delta x_2$$

$$\frac{\cancel{\bar{F}_e}}{k_e} = \frac{\cancel{\bar{F}_1}}{k_1} + \frac{\cancel{\bar{F}_2}}{k_2} \quad ; \quad \bar{F}_e = \bar{F}_1 = \bar{F}_2$$

$$\frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2}$$