AKADEMIA EDUKACYJNO - INŻYNIERSKA

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MATURA STUDIA PRAKTYKA STAŻ PRACA BIZNES PRZEMYSŁ STEAM

KOMPLEKSOWE WSPARCIE EDUKACYJNE **ON – LINE** NA KAŻDYM ETAPIE KSZTAŁCENIA INŻYNIERSKIEGO

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MECHANIKA III

"ZASADA d'ALEMBERTA"

PROJEKT

Oblicz:

- przyspieszenie liniowe i kątowe
- siłę w cięgnie

dla układu brył sztywnych.

DANE:

1) $m_1, \mu_1 = OK$
2) $m_2, r_2 = OK$
3) $m_3, r_3 = OK$

SZUKAM:

1) $a_1 = ?$
2) $\varepsilon_2 = ?$
3) $a_3, \varepsilon_3 = ?$

$S = ?$

M3

D: m_1, m_2, m_3 = ✓

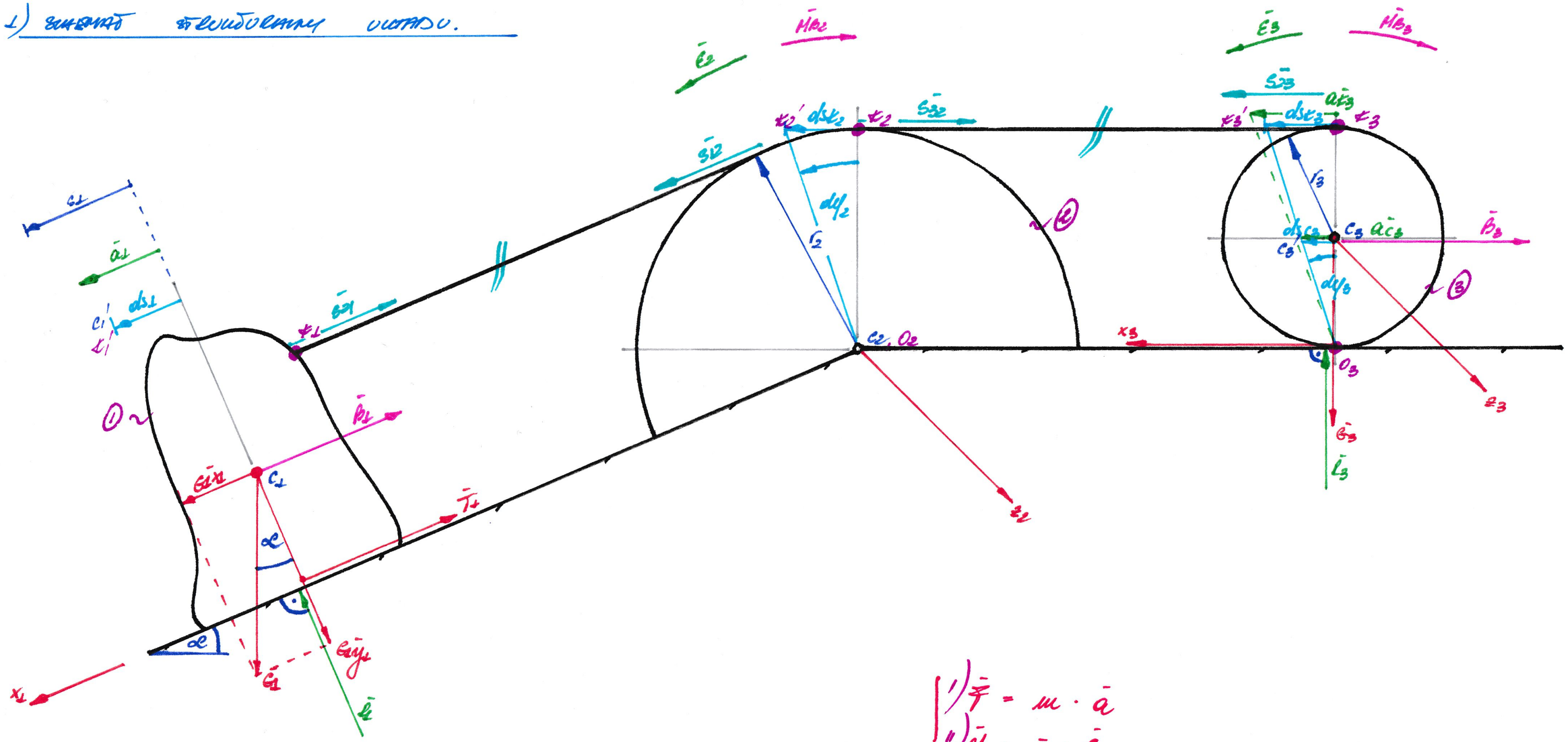
r_2, r_3 = ✓

φ_1 = ✓

SE: $\bar{a}_1, \bar{\varepsilon}_2, \bar{a}_3, \bar{\varepsilon}_3 = ?$

$S = ?$

1) SCHÉMA [illegible] [illegible].



$$\begin{cases} \text{I)}\ \bar{F} = m \cdot \bar{a} \\ \text{II)}\ \bar{M} = I \cdot \bar{\varepsilon} \\ \text{III)}\ dW = \bar{F} \cdot ds + M \cdot d\varphi = 0 \end{cases}$$

2) [illegible].

$$dW = F \cdot ds + M \cdot d\varphi = 0$$

$$\begin{cases} {}^{1)}\ G_1 x_1 \cdot ds_1 - T_1 \cdot ds_1 - B_1 \cdot ds_1 \\ {}^{2)}\ - M_{B2} \cdot d\varphi_2 \\ {}^{3)}\ - B_3 \cdot ds_3 - M_{B3} \cdot d\varphi_3 \end{cases}$$

(+)

2) $M_{B2} = I_2 \cdot \varepsilon_2 = \frac{1}{2} m_2 r_2^2 \cdot \varepsilon_2$

3) $B_3 = m_3 \cdot a_3$

$M_{B3} = I_3 \cdot \varepsilon_3 = \frac{1}{2} m_3 r_3^2 \cdot \varepsilon_3$

$\Rightarrow$ ${}^{1)}\ m_1 g \sin\alpha \cdot ds_1 - m_1 g \mu_1 \cos\alpha \cdot ds_1 - m_1 a_1 \cdot ds_1$ ${}^{2)}\ - \frac{1}{2} m_2 r_2^2 \cdot \varepsilon_2 \cdot d\varphi_2$ ${}^{3)}\ - m_3 a_3 \cdot ds_3 - \frac{1}{2} m_3 r_3^2 \cdot \varepsilon_3 \cdot d\varphi_3 = 0$

$${}^{1)}\ G_1 x_1 \cdot ds_1 - T_1 \cdot ds_1 - B_1 \cdot ds_1 \ {}^{2)} - M_{B2} \cdot d\varphi_2 \ {}^{3)} - B_3 \cdot ds_3 - M_{B3} \cdot d\varphi_3 = 0$$

1) $\frac{G_1 x_1}{G_1} = \sin\alpha \Rightarrow G_1 x_1 = m_1 g \sin\alpha$

$\frac{G_1 y_1}{G_1} = \cos\alpha \Rightarrow G_1 y_1 = m_1 g \cos\alpha$

$\mu = \frac{F_T}{F_N} \Rightarrow T_1 = \mu_1 \cdot N_1 = \mu_1 \cdot G y_1 = m_1 g \mu_1 \cos\alpha$

$B_1 = m_1 a_1$

$$\begin{cases} {}^{I)}\ x = \varphi \cdot r \\ {}^{II)}\ v = \omega \cdot r \\ {}^{III)}\ a^{\tau} = \varepsilon \cdot r \end{cases}$$

$q = a_1$)

$a_1 = \varepsilon_2 \cdot r_2 \Rightarrow \varepsilon_2 = \frac{a_1}{r_2}$

$$\begin{cases} {}^{I)}\ a_{\tau 2} = a_1 \\ {}^{II)}\ a_{\tau 2} = \varepsilon_2 \cdot r_2 \end{cases}$$

$$\begin{cases} {}^{I)}\ a_{\tau 3} = a_{\tau 2} = a_1 \\ {}^{II)}\ a_{\tau 3} = \varepsilon_3 \cdot 2r_3 \end{cases}$$

(+)

$a_{\tau 3} = a_1 = \varepsilon_3 \cdot 2r_3$

$\Rightarrow \varepsilon_3 = \frac{a_1}{2r_3}$

$a_3 = \varepsilon_3 \cdot r_3 = \frac{a_1}{2r_3} \cdot r_3 = \frac{1}{2} a_1$

$${}^{1)}\ m_1 g \sin\alpha \cdot ds_1 - m_1 g \mu_1 \cos\alpha \cdot ds_1 - m_1 a_1 \cdot ds_1 \ {}^{2)} - \frac{1}{2} m_2 r_2^2 \cdot \frac{a_1}{r_2} \cdot d\varphi_2 \ {}^{3)} - m_3 \cdot \frac{1}{2} a_1 \cdot ds_3 - \frac{1}{2} m_3 r_3^2 \cdot \frac{a_1}{2r_3} \cdot d\varphi_3 = 0$$

-2-

$x = y \cdot r$

$q = ds_1$

$ds_1 = d\varphi_2 \cdot r_2 \quad \Rightarrow \quad d\varphi_2 = \frac{ds_1}{r_2}$

1) $ds_{x_2} = ds_1$

2) $ds_{x_2} = d\varphi_1 \cdot r_2$

1) $ds_{x_3} = ds_{x_2} = ds_1$

2) $ds_{x_3} = d\varphi_3 \cdot r_3$

$\oplus$

$ds_{x_3} = ds_1 = d\varphi_3 \cdot r_3$

$\Rightarrow \quad d\varphi_3 = \frac{ds_1}{r_3}$

$ds_3 = d\varphi_3 \cdot r_3 = \frac{ds_1}{r_3} \cdot r_3 = \frac{1}{2} ds_1$

1) $m_1 g \sin\alpha \cdot ds_1 - m_1 g y_1 \cos\alpha \cdot ds_1 - m_1 a_1 \cdot ds_1 - 2) \frac{1}{2} m_2 r_2^2 \cdot \frac{a_1}{r_2} \cdot \frac{ds_1}{r_2} - 3) m_3 \cdot \frac{1}{2} a_1 \cdot \frac{1}{2} ds_1 - \frac{1}{2} m_3 r_3^2 \cdot \frac{a_1}{r_3} \cdot \frac{ds_1}{r_3} = 0$

$m_1 g \sin\alpha - m_1 g y_1 \cos\alpha = m_1 a_1 + \frac{1}{2} m_2 a_1 + \frac{1}{4} m_3 a_1 + \frac{1}{8} m_3 a_1$

$\Rightarrow$ 1) $a_1(t) = \frac{m_1 g (\sin\alpha - y_1 \cos\alpha)}{(m_1 + \frac{1}{2} m_2 + \frac{3}{8} m_3)}$

2) $\varepsilon_2(t) = \frac{a_1(t)}{r_2}$

3) $a_3(t) = \frac{1}{2} a_1(t)$

$\varepsilon_3(t) = \frac{a_1(t)}{r_3}$

2) описываем.

$$\frac{d}{dt}\left(\frac{dL}{d\dot{q}}\right) - \frac{dL}{dq} = Q \quad \Rightarrow \quad \frac{d}{dt}\left(\frac{dL}{d\dot{x}_1}\right) - \frac{dL}{dx_1} = 0$$

$q = x_1$

1) I) $E_{k1} = \frac{1}{2} m_1 \dot{x}_1^2$

II) $E_{p1}(x_1) = (G_{x1} - T_1)(s_1 - x_1)$

2) I) $E_{k2} = \frac{1}{2} I_2 \dot{\varphi}_2^2$

II) $E_{p2} = 0$

3) I) $E_{k3} = \frac{1}{2} m_3 \dot{x}_3^2 + \frac{1}{2} I_3 \dot{\varphi}_3^2$

II) $E_{p3} = 0$

$$L = E_k - E_p$$

$$L = (E_{k1} + E_{k2} + E_{k3}) - (E_{p1} + E_{p2} + E_{p3})$$

$$L = \left(\frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} I_2 \dot{\varphi}_2^2 + \frac{1}{2} m_3 \dot{x}_3^2 + \frac{1}{2} I_3 \dot{\varphi}_3^2\right) - (G_{x1} - T_1)(s_1 - x_1)$$

1) $\frac{G_{x1}}{G_1} = \sin\alpha \Rightarrow G_{x1} = m_1 g \sin\alpha$

$\frac{G_{y1}}{G_1} = \cos\alpha \Rightarrow G_{y1} = m_1 g \cos\alpha$

$f = \frac{F_T}{F_N} \Rightarrow T_1 = \mu_1 \cdot R_1 = \mu_1 \cdot G_{y1} = m_1 g \mu_1 \cos\alpha$

2) $I_2 = \frac{1}{2} m_2 r_2^2$

3) $I_3 = \frac{1}{2} m_3 r_3^2$

$$L = \left(\frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} \cdot \frac{1}{2} m_2 r_2^2 \dot{\varphi}_2^2 + \frac{1}{2} m_3 \dot{x}_3^2 + \frac{1}{2} \cdot \frac{1}{2} m_3 r_3^2 \dot{\varphi}_3^2\right) - (m_1 g \sin\alpha - m_1 g \mu_1 \cos\alpha)(s_1 - x_1)$$

$$L = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{4} m_2 r_2^2 \dot{\varphi}_2^2 + \frac{1}{2} m_3 \dot{x}_3^2 + \frac{1}{4} m_3 r_3^2 \dot{\varphi}_3^2 + m_1 g x_1 \sin\alpha - m_1 g \mu_1 x_1 \cos\alpha - m_1 g s_1 \sin\alpha + m_1 g \mu_1 s_1 \cos\alpha$$

$$L = 1)\ \frac{1}{2} m_1 \dot{x}_1^2 + 2)\ \frac{1}{4} m_2 r_2^2 \dot{\varphi}_2^2 + 3)\ \frac{1}{2} m_3 \dot{x}_3^2 + \frac{1}{4} m_3 r_3^2 \dot{\varphi}_3^2 + 4)\ m_1 g x_1 \sin\alpha - m_1 g y_1 x_1 \cos\alpha - m_1 g z_1 \sin\alpha + m_1 g y_1 z_1 \cos\alpha$$

$q = x_1$

$$x_1 = x_{t1} = x_{t1} = x_1$$

$$\begin{cases} I) & x_{t2} = x_{t1} = x_1 \\ II) & x_{t2} = \varphi_2 \cdot r_2 \end{cases} \quad (+)$$

$$x_{t2} = x_1 = \varphi_2 \cdot r_2 \Rightarrow \varphi_2 = \frac{x_1}{r_2}$$

$$\begin{cases} I) & x_{t3} = x_{t2} = x_1 \\ II) & x_{t3} = \varphi_3 \cdot 2r_3 \end{cases} \quad (+)$$

$$x_{t3} = x_1 = \varphi_3 \cdot 2r_3 \Rightarrow \varphi_3 = \frac{x_1}{2r_3}$$

$$x_3 = \varphi_3 \cdot r_3 = \frac{x_1}{2r_3} \cdot r_3 = \frac{1}{2} x_1$$

$$\frac{d}{dt}\left(\frac{dL}{d\dot{x}_1}\right) - \frac{dL}{dx_1} = 0$$

$$\left(m_1 + \frac{1}{2} m_2 + \frac{3}{8} m_3\right) \ddot{x}_1 - m_1 g \sin\alpha + m_1 g y_1 \cos\alpha = 0$$

$$1)\ \ddot{x}_1(t) = \frac{m_1 (\sin\alpha - y_1 \cos\alpha)}{\left(m_1 + \frac{1}{2} m_2 + \frac{3}{8} m_3\right)} g$$

$$2)\ \ddot{\varphi}_2(t) = \frac{\ddot{x}_1(t)}{r_2}$$

$$3)\ \ddot{x}_3(t) = \frac{1}{2} \ddot{x}_1(t)$$

$$\ddot{\varphi}_3(t) = \frac{\ddot{x}_1(t)}{2r_3}$$

$$L = 1)\ \frac{1}{2} m_1 \dot{x}_1^2 + 2)\ \frac{1}{4} m_2 r_2^2 \cdot \frac{\dot{x}_1^2}{r_2^2} + 3)\ \frac{1}{2} m_3 \cdot \frac{1}{4} \dot{x}_1^2 + \frac{1}{4} m_3 r_3^2 \cdot \frac{\dot{x}_1^2}{4r_3^2} + 4)\ m_1 g x_1 \sin\alpha - m_1 g y_1 x_1 \cos\alpha - m_1 g z_1 \sin\alpha + m_1 g y_1 z_1 \cos\alpha$$

$$L = 1)\ \frac{1}{2} m_1 \dot{x}_1^2 + 2)\ \frac{1}{4} m_2 \dot{x}_1^2 + 3)\ \frac{3}{16} m_3 \dot{x}_1^2 + 4)\ m_1 g x_1 \sin\alpha - m_1 g y_1 x_1 \cos\alpha - m_1 g z_1 \sin\alpha + m_1 g y_1 z_1 \cos\alpha$$

$$\frac{dL}{dx_1} = m_1 g \sin\alpha - m_1 g y_1 \cos\alpha$$

$$\frac{dL}{d\dot{x}_1} = m_1 \dot{x}_1 + \frac{1}{2} m_2 \dot{x}_1 + \frac{3}{8} m_3 \dot{x}_1$$

$$\frac{d}{dt}\left(\frac{dL}{d\dot{x}_1}\right) = m_1 \ddot{x}_1 + \frac{1}{2} m_2 \ddot{x}_1 + \frac{3}{8} m_3 \ddot{x}_1$$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