AKADEMIA EDUKACYJNO - INŻYNIERSKA

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MATURA STUDIA PRAKTYKA STAŻ PRACA BIZNES PRZEMYSŁ STEAM

KOMPLEKSOWE WSPARCIE EDUKACYJNE **ON – LINE** NA KAŻDYM ETAPIE KSZTAŁCENIA INŻYNIERSKIEGO

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MECHANIKA III

"RÓWNANIA LA GRANGE'A II RODZAJU"

PROJEKT

Oblicz:

- równanie ruchu
- częstość drgań własnych

dla układu mechanicznego.

DANE:

1) m_1, l_1 = *OK*
2) m_2 = *OK*
3) c_3 = *OK*
4) c_4 = *OK*

SZUKAM:

$\varphi_1(t)$, ω_0 = *?*

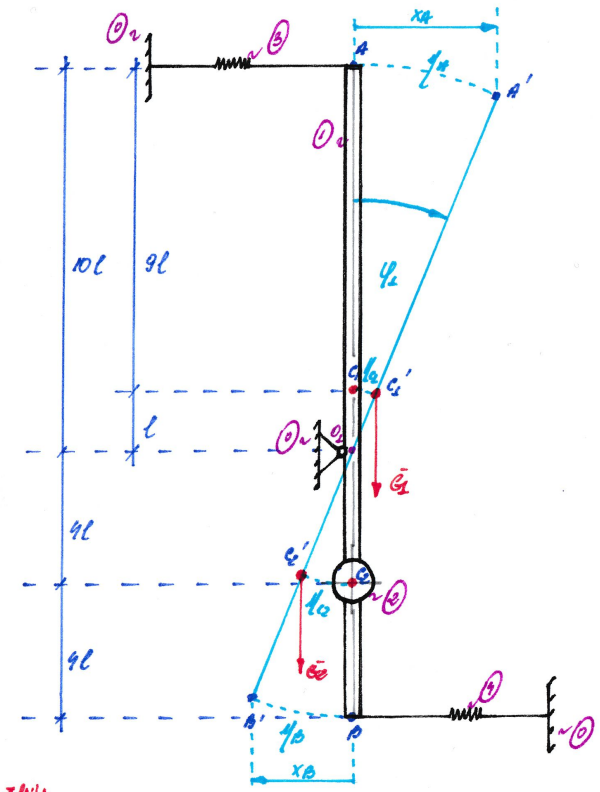
Nº3

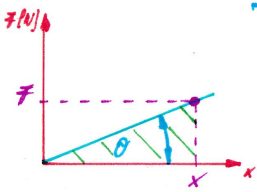
D: m_1, m_2, c_3, c_4, l = ✓

SZ: $\varphi(t) = ?$

$\omega_0 = ?$

1) SCHEMAT STEROWALNEGO UKŁADU.





$\Rightarrow N = \frac{1}{2} T x = \frac{1}{2} c x^2$

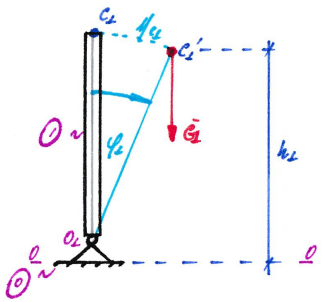
2) OBLICZENIA.

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = Q$

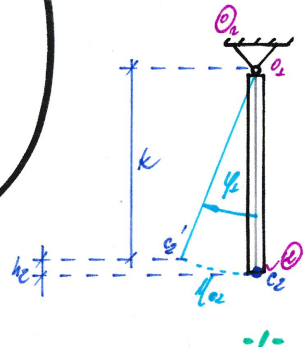
$q = \varphi_1$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}_1}\right) - \frac{\partial L}{\partial \varphi_1} = 0$

$L = E_k - E_p \Rightarrow$



$\frac{h_1}{|O_1C_1|} = \cos\varphi_1 \Rightarrow h_1 = |O_1C_1| \cos\varphi_1$



$h_2 = |O_1C_2| - k$

$\frac{k}{|O_1C_2|} = \cos\varphi_1 \Rightarrow k = |O_1C_2| \cos\varphi_1$

$\Rightarrow h_2 = |O_1C_2| - |O_1C_2| \cos\varphi_1 = |O_1C_2|(1 - \cos\varphi_1)$

1) I) $E_{k1} = \frac{1}{2} I_1^{(O_1)} \dot{\varphi}_1^2$

II) $E_{p1} = m_1 g h_1$

2) I) $E_{k2} = \frac{1}{2} I_2^{(O_2)} \dot{\varphi}_1^2$

II) $E_{p2} = m_2 g h_2$

3) I) $E_{k3} = 0$

II) $E_{p3} = \frac{1}{2} c_3 x_3^2$

4) I) $E_{k4} = 0$

II) $E_{p4} = \frac{1}{2} c_4 x_4^2$

$$L = E_k - E_p$$

$$L = (E_{k1} + E_{k2} + E_{k3} + E_{k4}) - (E_{p1} + E_{p2} + E_{p3} + E_{p4})$$

$$L = \left(\frac{1}{2} I_1^{(O_1)} \dot{\varphi}_1^2 + \frac{1}{2} I_2^{(O_1)} \dot{\varphi}_1^2\right) - \left(m_1 g h_1 + m_2 g h_2 + \frac{1}{2} c_3 x_3^2 + \frac{1}{2} c_4 x_4^2\right)$$

$$L = \frac{1}{2} I_1^{(O_1)} \dot{\varphi}_1^2 + \frac{1}{2} I_2^{(O_1)} \dot{\varphi}_1^2 - m_1 g l \cos\varphi_1 - m_2 g \cdot 4l (1 - \cos\varphi_1) - \frac{1}{2} c_3 x_3^2 - \frac{1}{2} c_4 x_4^2$$

$$L = \frac{1}{2} I_1^{(O_1)} \dot{\varphi}_1^2 + \frac{1}{2} I_2^{(O_1)} \dot{\varphi}_1^2 - m_1 g l \cos\varphi_1 + 4 m_2 g l \cos\varphi_1 - 4 m_2 g l - \frac{1}{2} c_3 x_3^2 - \frac{1}{2} c_4 x_4^2$$

$$I_1^{(O_1)} = I_1^{(C_1)} + m_1 (l_1)^2$$

$$I_1^{(O_1)} = \frac{1}{12} m_1 (18l)^2 + m_1 l^2$$

$$I_1^{(O_1)} = \frac{324}{12} m_1 l^2 + m_1 l^2$$

$$I_1^{(O_1)} = 27 m_1 l^2 + m_1 l^2 - 28 m_1 l^2$$

$$I_2^{(O_1)} = m_2 (l_2)^2$$

$$I_2^{(O_1)} = m_2 (4l)^2 = 16 m_2 l^2$$

$$x = \varphi \cdot l$$

$$\begin{cases} x_3 = \varphi_1 \cdot 10l \\ x_4 = \varphi_1 \cdot 8l \end{cases}$$

$$\begin{cases} x_3 = 10\varphi_1 l \\ x_4 = 8\varphi_1 l \end{cases}$$

$$L = \frac{1}{2} \cdot 28 m_1 l^2 \cdot \dot{\varphi}_1^2 + \frac{1}{2} \cdot 16 m_2 l^2 \cdot \dot{\varphi}_1^2 - m_1 g l \cos\varphi_1 + 4 m_2 g l \cos\varphi_1 - 4 m_2 g l - \frac{1}{2} c_3 \cdot (10\varphi_1 l)^2 - \frac{1}{2} c_4 \cdot (8\varphi_1 l)^2$$

$$L = 14 m_1 l^2 \dot{\varphi}_1^2 + 8 m_2 l^2 \dot{\varphi}_1^2 - m_1 g l \cos\varphi_1 + 4 m_2 g l \cos\varphi_1 - 4 m_2 g l - 50 c_3 l^2 \varphi_1^2 - 32 c_4 l^2 \varphi_1^2$$

$$L = 14 m_1 l^2 \dot{\varphi}_1^2 + 8 m_2 l^2 \dot{\varphi}_1^2 - 50 c_3 l^2 \varphi_1^2 - 32 c_4 l^2 \varphi_1^2 - m_1 g l \cos\varphi_1 + 4 m_2 g l \cos\varphi_1 - 4 m_2 g l$$

i) $\frac{\partial L}{\partial \varphi_1} = -100 c_3 l^2 \varphi_1 - 64 c_4 l^2 \varphi_1 + m_1 g l \sin\varphi_1 - 4 m_2 g l \sin\varphi_1$

ii) $\frac{\partial L}{\partial \dot{\varphi}_1} = 28 m_1 l^2 \dot{\varphi}_1 + 16 m_2 l^2 \dot{\varphi}_1$

iii) $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}_1}\right) = 28 m_1 l^2 \ddot{\varphi}_1 + 16 m_2 l^2 \ddot{\varphi}_1$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}_1}\right) - \frac{\partial L}{\partial \varphi_1} = 0$$

$$(28 m_1 l^2 + 16 m_2 l^2)\ddot{\varphi}_1 - (m_1 g l - 4 m_2 g l - 100 c_3 l^2 - 64 c_4 l^2)\varphi_1 = 0$$

$$(28 m_1 l^2 + 16 m_2 l^2)\ddot{\varphi}_1 + (4 m_2 g l - m_1 g l + 100 c_3 l^2 + 64 c_4 l^2)\varphi_1 = 0$$

$$\ddot{\varphi}_1 + \frac{(4 m_2 g - m_1 g + 100 c_3 l + 64 c_4 l)\cancel{l}}{(28 m_1 + 16 m_2)\cancel{l^2}}\varphi_1 = 0$$

$$\ddot{\varphi}_1(t) + \underbrace{\frac{(4 m_2 g - m_1 g + 100 c_3 l + 64 c_4 l)}{(28 m_1 + 16 m_2) l}}_{\omega_0^2}\varphi_1(t) = 0 \quad \Rightarrow \quad \omega_0 = \sqrt{\frac{(4 m_2 g - m_1 g + 100 c_3 l + 64 c_4 l)}{(28 m_1 + 16 m_2) l}}$$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