SERWIS EDUKACYJNO - INŻYNIERSKI

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MATURA STUDIA PRAKTYKA PRACA

KOMPLEKSOWE WSPARCIE EDUKACYJNE NA KAŻDYM ETAPIE KSZTAŁCENIA INŻYNIERSKIEGO

Matematyka ; Fizyka ; Algebra z geometrią analityczną ; Analiza matematyczna I, II, III ; Mechanika I, II, III ; Mechanika płynów ; Mechanika analityczna ; Mechanika kwantowa ; Mechanika Techniczna ; Wytrzymałość materiałów I, II, III ; Równania różniczkowe ; PKM I, II ; Podstawy konstrukcji maszyn ; TMM ; Teoria mechanizmów i manipulatorów ; AiSUK ; Analiza i synteza układów kinematycznych ; PPM ; Podstawy projektowania mechanizmów (maszyn) ; PPŚT ; Podstawy projektowania środków transportu ; Manipulatory ; Automatyka i robotyka ; Synteza mechanizmów ; Modelowanie układów wieloczłonowych ; Grafika inżynierska 2D i 3D ; maszyny CNC ; konsultacje prac inżynierskich i magisterskich kierunków studiów technicznych ; współpraca z przemysłem.

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MECHANIKA II

DYNAMIKA UKŁADU BRYŁ SZTYWNYCH

PROJEKT

Dla zadanego układu brył sztywnych, połączonego jak na rysunku poniżej znaleźć dla drogi S_1, przebytej przez ciało 1:

1) Parametry kinematyczne:

- drogi liniowe i kątowe każdego członu.
- prędkości liniowe i kątowe każdego członu.
- przyspieszenia liniowe i kątowe każdego członu.

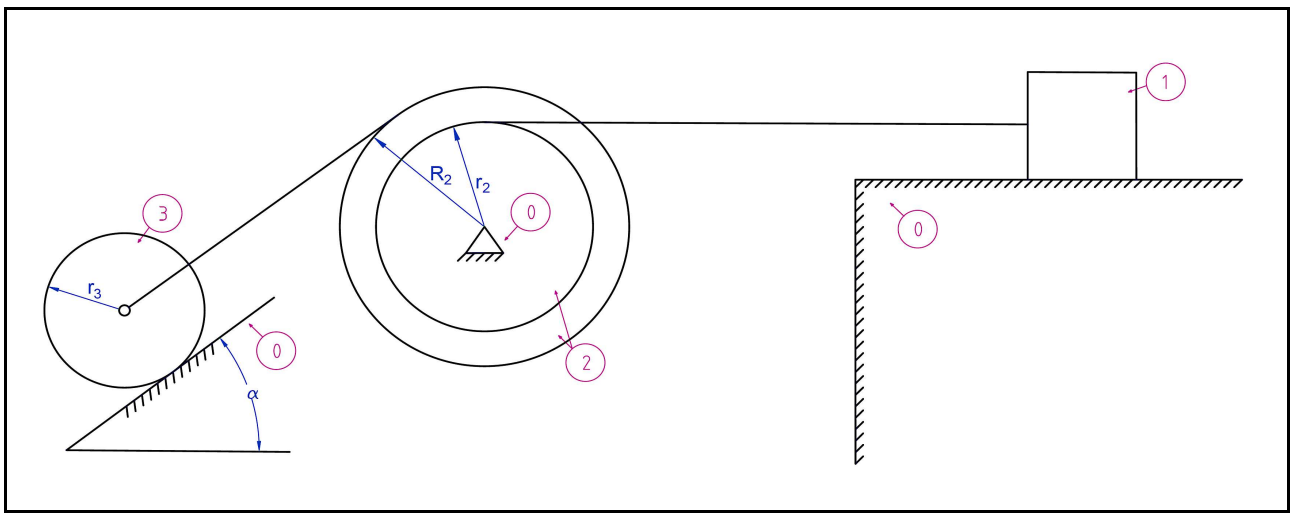
2) Parametry dynamiczne:

- siłę napięcia w cięgnach, łączących poszczególne człony.

DANE

- masy wszystkich członów.
- promienie tarcz kołowych.
- współczynnik tarcia ślizgowego i tocznego.
- kąt nachylenia równi pochyłej.
- droga, jaką przebyło ciało 1.

Założyć nieważkość i bezmasowość cięgien.



Rys. 1. Dynamika układu brył sztywnych.

ZAD. 01

DANE:

$m_1 = 1$ [kg]

$m_2 = 3$ [kg]

$M_2 = 4$ [kg]

$m_3 = 1$ [kg]

$\mu_1 = 0{,}1$

$\alpha = 36°$

$r_2 = 0{,}3$ [m]

$R_2 = 0{,}4$ [m]

$r_3 = 0{,}1$ [m]

$f_3 = 5 \cdot 10^{-5}$ [m]

$S_1 = 1{,}2$ [m]

SZUKANE:

$\bar{s}_3$; $\bar{\varphi}_2 = ?$

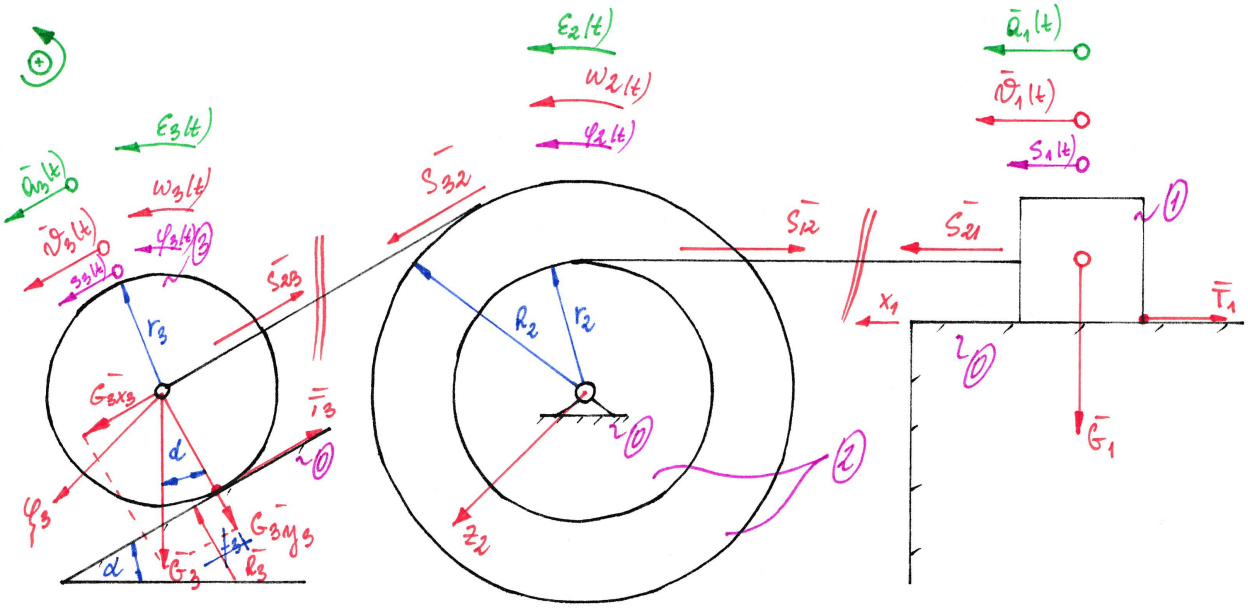
$\bar{v}_1$; $\bar{v}_3 = ?$

$\bar{\omega}_2$; $\bar{\omega}_3 = ?$

$\bar{a}_1$; $a_3 = ?$

ε_2 ; $\varepsilon_3 = ?$

1) SCHEMAT KINEMATYCZNY MECHANIZMU:



2) RUCH $x_1(t)$:

$$\bar{F} = m \cdot \bar{a}$$

$$\bar{M} = \bar{I} \cdot \bar{\varepsilon}$$

$$\Downarrow$$

$$\begin{cases} \sum F_{ix_1} = m_1 \cdot \ddot{x}_1 \\ \sum M_{iz_2} = I_{z_2} \cdot \ddot{\varphi}_2 \\ \sum F_{ix_3} = m_3 \cdot \ddot{x}_3 \\ \sum M_{i\varphi_3} = I_{\varphi_3} \cdot \ddot{\varphi}_3 \end{cases} \rightarrow \begin{cases} m_1 \ddot{x}_1 = \bar{S}_{21} - \bar{T}_1 \\ \bar{I}_{z_2} \cdot \ddot{\varphi}_2 = \bar{S}_{32} \cdot R_2 - \bar{S}_{12} \cdot r_2 \\ m_3 \cdot \ddot{x}_3 = G_{3x_3} - \bar{T}_3 - \bar{S}_{23} \\ \bar{I}_{\varphi_3} \cdot \ddot{\varphi}_3 = \bar{T}_3 \cdot r_3 - \bar{R}_3 \cdot t_3 \end{cases}$$

$$\bar{S}_{12} = \bar{S}_{21}$$

$$\bar{S}_{32} = \bar{S}_{23}$$

$$G_1 = m_1 g$$

$$T_1 = \mu_1 \cdot G_1 \Rightarrow \bar{T}_1 = \mu_1 m_1 g$$

$$I_{z_2} = \frac{1}{2}\left(m_2 r_2^2 + m_2 R_2^2\right)$$

$$\frac{G_{3x_3}}{G_3} = \sin\alpha \Rightarrow G_{3x_3} = m_3 g \sin\alpha$$

$$\frac{G_{3y_3}}{G_3} = \cos\alpha \Rightarrow G_{3y_3} = m_3 g \cos\alpha$$

$$I_{\varphi_3} = \frac{1}{2} m_3 r_3^2$$

$$\Rightarrow \begin{cases} m_1 \cdot \ddot{x}_1 = S_{21} - \mu_1 m_1 g \\ \frac{1}{2}\left(m_2 r_2^2 + m_2 R_2^2\right) \cdot \ddot{\varphi}_2 = S_{23} \cdot R_2 - S_{21} \cdot r_2 \\ m_3 \cdot \ddot{x}_3 = m_3 g \sin\alpha - T_3 - S_{23} \\ \frac{1}{2} m_3 r_3^2 \cdot \ddot{\varphi}_3 = T_3 \cdot r_3 - m_3 g f_3 \cos\alpha \end{cases}$$

$x_1(t) = ?$

$$\ddot{x}_1 = \ddot{\varphi}_2 \cdot r_2 \Rightarrow \ddot{\varphi}_2 = \frac{\ddot{x}_1}{r_2}$$

$$\ddot{x}_3 = \ddot{\varphi}_2 \cdot R_2 \Rightarrow \ddot{x}_3 = \frac{\ddot{x}_1}{r_2} \cdot R_2$$

$$\ddot{x}_3 = \ddot{\varphi}_3 \cdot r_3 \Rightarrow \ddot{\varphi}_3 = \frac{\ddot{x}_3}{r_3} = \frac{\ddot{x}_1}{r_2 \cdot r_3} \cdot R_2$$

$$\Rightarrow \begin{cases} 1)\ m_1 \cdot \ddot{x}_1 = S_{21} - \mu_1 m_1 g \\ 2)\ \frac{1}{2}\left(m_2 r_2^2 + m_2 R_2^2\right) \cdot \frac{\ddot{x}_1}{r_2} = S_{23} \cdot R_2 - S_{21} \cdot r_2 \\ 3)\ m_3 \cdot \frac{\ddot{x}_1}{r_2} \cdot R_2 = m_3 g \sin\alpha - T_3 - S_{23} \\ 4)\ \frac{1}{2} m_3 r_3^2 \cdot \frac{\ddot{x}_1}{r_2 \cdot r_3} \cdot R_2 = T_3 \cdot r_3 - m_3 g f_3 \cos\alpha \end{cases}$$

1/ $$S_{21} = m_1 \ddot{x}_1 + \mu_1 m_1 g$$

$$\Rightarrow \quad S_{21} = \ddot{x}_1 + 0{,}981$$

2/ $$\frac{1}{2}\left(m_2 r_2^2 + M_2 R_2^2\right) \cdot \frac{\ddot{x}_1}{r_2} = S_{23} \cdot R_2 - \left(\ddot{x}_1 + 0{,}981\right) \cdot r_2$$

$$\Rightarrow \quad \frac{1}{2}\left(3 \cdot (0{,}3)^2 + 4 \cdot (0{,}4)^2\right) \cdot \frac{\ddot{x}_1}{0{,}3} = S_{23} \cdot 0{,}4 - 0{,}3\left(\ddot{x}_1 + 0{,}981\right)$$

$$\frac{1}{2}\left(0{,}27 + 0{,}64\right) \cdot \frac{\ddot{x}_1}{0{,}3} = 0{,}4 S_{23} - 0{,}3 \ddot{x}_1 - 0{,}294$$

$$1{,}517 \ddot{x}_1 = 0{,}4 S_{23} - 0{,}3 \ddot{x}_1 - 0{,}294$$

$$S_{23} = \frac{1{,}517 \ddot{x}_1 + 0{,}3 \ddot{x}_1 + 0{,}294}{0{,}4} = \frac{1{,}817 \ddot{x}_1 + 0{,}294}{0{,}4}$$

$$\Rightarrow \quad S_{23} = 4{,}543 \ddot{x}_1 + 0{,}735$$

3/ $T_3 = m_3 g \sin\alpha - S_{23} - m_3 \frac{\ddot{x}_1}{r_2} \cdot R_2 =$

$= 1 \cdot 9{,}81 \cdot \sin 36^\circ - 4{,}543\ddot{x}_1 - 0{,}735 - 1 \cdot \frac{\ddot{x}_1}{0{,}13} \cdot 0{,}4 =$

$= 5{,}766 - 4{,}543\ddot{x}_1 - 0{,}735 - 1{,}333\ddot{x}_1 =$

$= -5{,}876\ddot{x}_1 + 5{,}031$

$\Rightarrow \boxed{T_3 = -5{,}876\ddot{x}_1 + 5{,}031}$

4/ $\frac{1}{2} m_3 r_3^2 \cdot \frac{\ddot{x}_1}{r_2 \cdot r_3} \cdot R_2 = T_3 \cdot r_3 - m_3 g f_3 \cos\alpha$

$\frac{1}{2} \cdot 1 \cdot (0{,}1)^2 \cdot \frac{\ddot{x}_1}{0{,}13 \cdot 0{,}1} \cdot 0{,}4 = (-5{,}876\ddot{x}_1 + 5{,}031) \cdot 0{,}1 - 1 \cdot 9{,}81 \cdot 5 \cdot 10^{-5} \cdot \cos 36^\circ$

$\frac{4 \cdot 10^{-3}}{0{,}06}\ddot{x}_1 = -0{,}5876\ddot{x}_1 + 0{,}5031 - 39{,}682 \cdot 10^{-5}$

$0{,}066\ddot{x}_1 = -0{,}5876\ddot{x}_1 + 0{,}503$

$0{,}066\ddot{x}_1 + 0{,}5876\ddot{x}_1 = 0{,}503$

$0{,}654\ddot{x}_1 = 0{,}503$

$\ddot{x}_1 = \frac{0{,}503}{0{,}654} = 0{,}769$

$\Rightarrow \boxed{\ddot{x}_1 = 0{,}769}$

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$$\ddot{x}_1(t) = 0{,}769$$

$$\dot{x}_1(t) = 0{,}769\,t + C_1$$

$$x_1(t) = 0{,}385\,t^2 + C_1 t + C_2$$

$$\begin{cases} x_1(t=0) = x_0 \\ \dot{x}_1(t=0) = v_0 \end{cases} \Rightarrow \begin{cases} C_1 = v_0 \\ C_2 = x_0 \end{cases}$$

$$\Rightarrow \begin{cases} \ddot{x}_1(t) = 0{,}769 \\ \dot{x}_1(t) = 0{,}769\,t + v_0 \\ x_1(t) = 0{,}385\,t^2 + v_0 t + x_0 \end{cases}$$

$$\begin{cases} v_0 = 0 \\ x_0 = 0 \end{cases} \Rightarrow \begin{cases} \ddot{x}_1(t) = 0{,}769 \\ \dot{x}_1(t) = 0{,}769\,t \\ x_1(t) = 0{,}385\,t^2 \end{cases}$$

3) ANALIZA MATEMATYCZNA - PRĘDKOŚCI

T - CZAS TRWANIA RUCHU (s_1).

T:

$$x_1(T) = s_1$$

$$\Rightarrow 0{,}385\,T^2 = s_1$$

$$\Rightarrow T = +\sqrt{\frac{s_1}{0{,}385}} = +\sqrt{\frac{1{,}2}{0{,}385}} = +\sqrt{3{,}117}$$

$$\Rightarrow \boxed{T = +1{,}766\ [s]}$$

$v_1(T)$:

$$\dot{x}_1(T) = 0{,}769\,T = 0{,}769 \cdot 1{,}766 = 1{,}358$$

$$\Rightarrow \boxed{v_1(T) = 1{,}358\ [m/s]}$$

$\omega_2(\bar{t})$:

$$v = \omega \cdot R$$

$$v_1(\bar{t}) = \omega_2(\bar{t}) \cdot r_2$$

$$\omega_2(\bar{t}) = \frac{v_1(\bar{t})}{r_2}$$

$$\omega_2(\bar{t}) = \frac{1,358}{0,3} = 4,527$$

$$\omega_2(\bar{t}) = 4,527 \ [s^{-1}]$$

$v_3(\bar{t})$:

$$v_3(\bar{t}) = \omega_2(\bar{t}) \cdot R_2$$

$$v_3(\bar{t}) = 4,527 \cdot 0,4 = 1,811$$

$$v_3(\bar{t}) = 1,811 \ \left[\frac{m}{s}\right]$$

$\omega_3(T)$:

$$v_3(T) = \omega_3(T) \cdot r_3$$

$$\Rightarrow \boxed{\omega_3(T) = \frac{v_3(T)}{r_3}}$$

$$\Rightarrow \omega_3(T) = \frac{1,811}{0,1} = 18,11$$

$$\Rightarrow \boxed{\omega_3(T) = 18,11 \ (s^{-1})}$$

4) Analiza wielkości - przyśpieszenia:

$a_1(\bar{T})$:

$$\ddot{x}_1(t) = 0{,}76\,9$$

$$\Rightarrow \quad a_1(\bar{T}) = 0{,}76\,9 \quad \left[\frac{m}{s^2}\right]$$

$\varepsilon_2(\bar{T})$:

$$a = \varepsilon \cdot R$$

$$\Rightarrow \quad \varepsilon_2(\bar{T}) = \frac{a_1(\bar{T})}{r_2}$$

$$\Rightarrow \quad \varepsilon_2(\bar{T}) = \frac{0{,}76\,9}{0{,}5} = 2{,}56\,3$$

$$\Rightarrow \quad \varepsilon_2(\bar{T}) = 2{,}56\,3 \quad [s^{-2}]$$

$a_3(\tau)$:

$$a_3(\tau) = \varepsilon_2(\tau) \cdot R_2$$

$$\Rightarrow \quad a_3(\tau) = 2{,}563 \cdot 0{,}4 = 1{,}025$$

$$\Rightarrow \quad \boxed{a_3(\tau) = 1{,}025 \ \left[\frac{m}{s^2}\right]}$$

$\varepsilon_3(\tau)$:

$$a_3(\tau) = \varepsilon_3(\tau) \cdot r_3$$

$$\Rightarrow \quad \varepsilon_3(\tau) = \frac{a_3(\tau)}{r_3} = \frac{1{,}025}{0{,}1} = 10{,}25$$

$$\Rightarrow \quad \boxed{\varepsilon_3(\tau) = 10{,}25 \ [s^{-2}]}$$

5) ANALIZA KINEMATYCZNA - ROZWIĄZANIE:

$s_1(\tau)$:

$$x_1(\tau) = s_1 = 1{,}2 \ [m]$$

$\varphi_2(\tau)$:

$$s_1(\tau) = \varphi_2(\tau) \cdot r_2$$

$$\Rightarrow \varphi_2(\tau) = \frac{s_1(\tau)}{r_2} = \frac{1{,}2}{0{,}3} = 4$$

$$\Rightarrow \varphi_2(\tau) = 4 \ [rad.]$$

$s_3(\tau)$:

$$s_3(\tau) = \varphi_2(\tau) \cdot R_2$$

$$\Rightarrow s_3(\tau) = 4 \cdot 0{,}4 = 1{,}6$$

$$\Rightarrow s_3(\tau) = 1{,}6 \ [m]$$

$\varphi_3(\tau)$:

$$s_3(\tau) = \varphi_3(\tau) \cdot r_3$$

$$\Rightarrow \varphi_3(\tau) = \frac{s_3(\tau)}{r_3} = \frac{1{,}6}{0{,}1} = 16 \Rightarrow \varphi_3(\tau) = 16 \ [rad.]$$

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b) WAŁ W CIĘGNACH:

$$S_{21} = \ddot{x}_1(T) + 0{,}981$$

$$S_{23} = 4{,}543\,\ddot{x}_1(T) + 0{,}735$$

$$\ddot{x}_1(T) = 0{,}769 \quad \left[\frac{m}{s^2}\right]$$

$$\Rightarrow S_{21} = \ddot{x}_1(T) + 0{,}981 = 0{,}769 + 0{,}981 = 1{,}75$$

$$\Rightarrow S_{21} = 1{,}75 \ [N]$$

$$\Rightarrow S_{23} = 4{,}543\,\ddot{x}_1(T) + 0{,}735 = 4{,}543 \cdot 0{,}769 + 0{,}735 =$$

$$= 3{,}494 + 0{,}735 = 4{,}229$$

$$\Rightarrow S_{23} = 4{,}229 \ [N]$$

6) ENERGIA KINETYCZNA UKŁADU:

$$E_{kT} = \frac{1}{2} m v^2$$

$$E_{kR} = \frac{1}{2} I \omega^2$$

1)

$$E_{k1} = \frac{1}{2} m_1 v_1^2$$

$$\Rightarrow E_{k1} = \frac{1}{2} m_1 v_1^2 = \frac{1}{2} \cdot 1 \cdot v_1^2 = 0{,}5 v_1^2 \Rightarrow \boxed{E_{k1} = 0{,}5 v_1^2}$$

2)

$$E_{k2} = \frac{1}{2} I_{z2} \omega_2^2$$

$$\Rightarrow E_{k2} = \frac{1}{2} I_{z2} \cdot \omega_2^2 = \frac{1}{2} \left(\frac{1}{2} (m_2 r_2^2 + m_2 R_2^2) \right) \omega_2^2 =$$

$$= 0{,}25 \left(3 \cdot (0{,}3)^2 + 4 \cdot (0{,}4)^2 \right) \omega_2^2 = 0{,}25 \left(0{,}27 + 0{,}64 \right) \omega_2^2 = 0{,}228 \omega_2^2$$

$$v_1 = \omega_2 \cdot r_2 \Rightarrow \boxed{\omega_2 = \frac{v_1}{r_2}}$$

$$\Rightarrow E_{k2} = 0{,}228 \cdot \frac{v_1^2}{r_2^2} = \frac{0{,}228}{0{,}09} v_1^2$$

$$\Rightarrow \boxed{E_{k2} = 2{,}533 v_1^2}$$

3)

$$E_{k3} = \frac{1}{2} m_3 v_3^2 + \frac{1}{2} I_{\varphi 3} \omega_3^2$$

$$\Rightarrow E_{k3} = \frac{1}{2} m_3 v_3^2 + \frac{1}{2} \cdot \frac{1}{2} m_3 r_3^2 \omega_3^2 =$$

$$= 0{,}5 \cdot 1 \cdot v_3^2 + 0{,}125 \cdot 1 \cdot (0{,}1)^2 \cdot \omega_3^2 = 0{,}5 v_3^2 + 0{,}0025 \omega_3^2$$

$$v_3 = \omega_2 \cdot R_2 \Rightarrow \boxed{v_3 = \frac{R_2}{r_2} \cdot v_1}$$

$$v_3 = \omega_3 \cdot r_3 \Rightarrow \boxed{\omega_3 = \frac{v_3}{r_3} = \frac{R_2}{r_2 r_3} \cdot v_1}$$

$$\Rightarrow E_{k3} = 0{,}5 \cdot \left(\frac{R_2}{r_2}\right)^2 \cdot v_1^2 + 0{,}0025 \cdot \left(\frac{R_2}{r_2 r_3}\right)^2 \cdot v_1^2 =$$

$$= 0{,}5 \cdot \left(\frac{0{,}4}{0{,}3}\right)^2 \cdot v_1^2 + 0{,}0025 \cdot \left(\frac{0{,}4}{0{,}3 \cdot 0{,}1}\right)^2 \cdot v_1^2 =$$

$$= 0{,}5 \cdot \frac{0{,}16}{0{,}09} \cdot v_1^2 + 0{,}0025 \cdot \frac{0{,}16}{9 \cdot 10^{-4}} \cdot v_1^2 =$$

$$= 0{,}888 \cdot v_1^2 + 0{,}444 v_1^2$$

$$\Rightarrow \boxed{E_{k3} = 1{,}332 v_1^2}$$

$$\Rightarrow \quad E_{kR3} = E_{k1} + E_{k2} + E_{k3}$$

$$\Rightarrow \quad E_{kR3} = 0{,}1\,v_1^2 + 2{,}133\,v_1^2 + 1{,}332\,v_1^2 = 4{,}365\,v_1^2$$

$$\Rightarrow \quad E_{kR3} = 4{,}365\,v_1^2$$

b) praca sił czynnych:

$$W_T = F \cdot s$$

$$W_R = M \cdot \varphi$$

$$\Downarrow$$

1)

$$W_1 = \sum F_{ix_1} \cdot x_1$$

$$\Downarrow$$

$$W_1 = -T_1 \cdot s_1 = -\mu_1 m_1 g \cdot s_1 = -0{,}1 \cdot 1 \cdot 9{,}81 \cdot s_1 = -0{,}981 \cdot s_1$$

$$\Rightarrow \quad W_1 = -0{,}981\,s_1$$

2/

$$W_2 = \sum M_{iz_2} \cdot \varphi_2$$

$$\Downarrow$$

$$W_2 = 0$$

3/

$$W_3 = \sum F_{ix_3} \cdot x_3 + \sum M_{i\varphi_3} \cdot \varphi_3$$

$$\Downarrow$$

$$W_3 = G_{3x_3} \cdot s_3 - R_3 \cdot f_3 \cdot \varphi_3 = m_3 g \sin\alpha \cdot s_3 - m_3 g f_3 \cos\alpha \cdot \varphi_3 =$$

$$= \begin{cases} s_1 = \varphi_2 \cdot r_2 \Rightarrow \boxed{\varphi_2 = \frac{s_1}{r_2}} \\ s_3 = \varphi_2 \cdot R_2 = \frac{R_2}{r_2} \cdot s_1 \Rightarrow \boxed{s_3 = \frac{R_2}{r_2} \cdot s_1} \\ s_3 = \varphi_3 \cdot r_3 \Rightarrow \varphi_3 = \frac{s_3}{r_3} = \frac{R_2}{r_2 r_3} \cdot s_1 \\ \Rightarrow \boxed{\varphi_3 = \frac{R_2}{r_2 r_3} \cdot s_1} \end{cases}$$

$$\Rightarrow W_3 = m_3 g \sin\alpha \cdot \frac{l_2}{r_2} \cdot s_1 - m_3 g f_3 \cos\alpha \cdot \frac{l_2}{r_2 r_3} \cdot s_1 =$$

$$= 1 \cdot 9{,}81 \cdot \sin 56^\circ \cdot \frac{0{,}4}{0{,}3} s_1 - 1 \cdot 9{,}81 \cdot 5 \cdot 10^{-5} \cdot \cos 56^\circ \cdot \frac{0{,}4}{0{,}3 \cdot 0{,}1} s_1 =$$

$$= 7{,}688\, s_1 - \frac{15{,}873 \cdot 10^{-5}}{0{,}03} \cdot s_1 = 7{,}688\, s_1 - 529{,}1 \cdot 10^{-5} s_1 =$$

$$= 7{,}683\, s_1$$

$$\Rightarrow \boxed{W_3 = 7{,}683\, s_1}$$

$$\boxed{W_{123} = W_1 + W_2 + W_3}$$

$$W_{123} = -0{,}981\, s_1 + 0 + 7{,}683\, s_1 = 6{,}702\, s_1$$

$$\Rightarrow \boxed{W_{123} = 6{,}702\, s_1}$$

8) ZASADA RÓWNOWAŻNOŚCI PRACY I ENERGII KINETYCZNEJ:

* ANALIZA RUCHU PRZYSPIESZONEGO - PRZYKŁADY.

$$W = \Delta E_k$$

$$W_{123} = E_{k123}$$

$v_1(T)$:

$$6{,}702\, s_1 = 4{,}365\, v_1^2$$

$$v_1 = \sqrt{\frac{6{,}702\, s_1}{4{,}365}}$$

$$\Rightarrow \boxed{v_1 = \sqrt{1{,}535\, s_1}} \qquad \{ \boxed{s_1 = 1{,}2\ [m]} \}$$

$$\Rightarrow v_1 = \sqrt{1{,}535 \cdot 1{,}2} = \sqrt{1{,}842}$$

$$\Rightarrow v_1 = 1{,}357$$

$$\Rightarrow \boxed{v_1(T) = 1{,}357 \quad \left[\frac{m}{s}\right]}$$

$\omega_2(\tau)$:

$$v = \omega \cdot R$$

$$\downarrow$$

$$v_1(\tau) = \omega_2(\tau) \cdot r_2$$

$$\Rightarrow \quad \omega_2(\tau) = \frac{v_1(\tau)}{r_2} = \frac{1,357}{0,3} = 4,523$$

$$\Rightarrow \quad \omega_2(\tau) = 4,523 \ [s^{-1}]$$

$v_3(\tau)$:

$$v_3(\tau) = \omega_2(\tau) \cdot R_2$$

$$\downarrow$$

$$v_3(\tau) = 4,523 \cdot 0,4 = 1,810$$

$$\Rightarrow \quad v_3(\tau) = 1,810 \ \left[\frac{m}{s}\right]$$

$\omega_3(\tau)$:

$$v_3(\tau) = \omega_3(\tau) \cdot r_3$$

$$\Rightarrow \quad \omega_3(\tau) = \frac{v_3(\tau)}{r_3} = \frac{1,810}{0,1} = 18,1 \quad \Rightarrow \quad \omega_3(\tau) = 18,1 \ [s^{-1}]$$

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9) ANALIZA KINEMATYCZNA - RÓWNANIA; $x_1(t)$:

$\bar{T}$ - CZAS TRWANIA RUCHU.

T:

$$\begin{cases} \ddot{x}_1(t) = A \\ \dot{x}_1(t) = At + v_0 \\ x_1(t) = \frac{1}{2}At^2 + v_0 t + x_0 \end{cases}$$

$$\begin{cases} x_1(t=0) = 0 \\ \dot{x}_1(t=0) = 0 \end{cases} \Rightarrow \begin{cases} x_0 = 0 \\ v_0 = 0 \end{cases}$$

$$\Rightarrow \begin{cases} \ddot{x}_1(t) = A \\ \dot{x}_1(t) = At \\ x_1(t) = \frac{1}{2}At^2 \end{cases}$$

$$\begin{cases} x_1(t=\bar{T}) = s_1 \\ \dot{x}_1(t=\bar{T}) = v_1 \end{cases} \Rightarrow \begin{cases} \frac{1}{2}A\bar{T}^2 = s_1 \\ A\bar{T} = v_1 \end{cases} \Rightarrow \frac{1}{2}v_1\bar{T} = s_1$$

$$\Rightarrow \bar{T} = \frac{2s_1}{v_1} = \frac{2 \cdot 1{,}2}{1{,}357} = 1{,}769 \Rightarrow \bar{T} = 1{,}769\ (s)$$

$\varphi_2(\tau)$:

$$s = \varphi \cdot r$$

$$\downarrow$$

$$s_1(\tau) = \varphi_2(\tau) \cdot r_2$$

$$\Rightarrow \quad \varphi_2(\tau) = \frac{s_1(\tau)}{r_2} = \frac{1{,}2}{0{,}3} = 4$$

$$\Rightarrow \quad \boxed{\varphi_2(\tau) = 4 \ [\text{wart.}]}$$

$s_3(\tau)$:

$$s_3(\tau) = \varphi_2(\tau) \cdot R_2$$

$$\downarrow$$

$$s_3(\tau) = 4 \cdot 0{,}4 = 1{,}6$$

$$\Rightarrow \quad \boxed{s_3(\tau) = 1{,}6 \ [\text{m}]}$$

10) ANALIZA KINEMATYCZNA - PRZYKŁAD LICZBOWY:

$Q_1(T)$:

$$Q_1 = A$$

$$\Downarrow$$

$$v_1(t) = Q_1 T$$

$$\Rightarrow Q_1(T) = \frac{v_1(T)}{T} = \frac{1,357}{1,169} = 0,767$$

$$\Rightarrow Q_1(T) = 0,767 \left[\frac{m}{s^2}\right]$$

$$\star \quad \frac{1}{2} Q_1(T) \cdot T^2 = s_1(T)$$

$$\Rightarrow Q_1(T) = \frac{2 s_1(T)}{T^2} = \frac{2 \cdot 1,2}{(1,169)^2} = \frac{2,4}{3,129} = 0,767$$

$$\Rightarrow Q_1(T) = 0,767 \left[\frac{m}{s^2}\right]$$

$\varepsilon_2(\bar{r})$:

$$a = \varepsilon \cdot R$$

$$\Downarrow$$

$$a_1(\bar{r}) = \varepsilon_2(\bar{r}) \cdot r_2$$

$$\Downarrow$$

$$\Rightarrow \quad \varepsilon_2(\bar{r}) = \frac{a_1(\bar{r})}{r_2} = \frac{0{,}767}{0{,}13} = 2{,}557$$

$$\Rightarrow \quad \boxed{\varepsilon_2(\bar{r}) = 2{,}557 \ [s^{-2}]}$$

$a_3(\bar{r})$:

$$a_3(\bar{r}) = \varepsilon_2(\bar{r}) \cdot R_2$$

$$\Downarrow$$

$$a_3(\bar{r}) = 2{,}557 \cdot 0{,}4 = 1{,}023$$

$$\Rightarrow \quad \boxed{a_3(\bar{r}) = 1{,}023 \ \left[\frac{m}{s^2}\right]}$$

$\varepsilon_3(\bar{r})$:

$$a_3(\bar{r}) = \varepsilon_3(\bar{r}) \cdot r_3$$

$$\Downarrow$$

$$\varepsilon_3(\bar{r}) = \frac{a_3(\bar{r})}{r_3} = \frac{1{,}023}{0{,}1} = 10{,}23 \quad \Rightarrow \quad \boxed{\varepsilon_3(\bar{r}) = 10{,}23 \ [s^{-2}]}$$

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